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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

EDITED BY

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Mount Wilson Solar Observatory of the
Carnegie Institution of Washington

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THE
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THE EMISSIVE POWER OF TUNGSTEN FOR SHORT WAVE-LENGTHS

By E. O. HULBURT

1. *Introductory.*—The ratio of the energy per second per unit area emitted normally from a metal at a given temperature to that from a black body at the same temperature is termed the "emissive power" of the metal. If the distribution of energy in the region of short wave-lengths of the spectrum of black-body radiation is given by Wien's equation, the distribution of energy of the metal's radiation in this region is expressed by the formula

$$J = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}} \cdot \epsilon, \qquad (1)$$

where J is the energy per unit wave-length radiated per second per unit area at wave-length λ by the metal at temperature T, and ϵ is the emissive power of the metal for the λ and T in question. In general ϵ is a function of λ and T. The problem of ascertaining the distribution of spectral energy of the metal's radiation thus resolves itself into a determination of the values of the emissive power for each wave-length and temperature. In the present investigation the changes in the emissive power of tungsten have been determined throughout the region of the spectrum from 3400 A to

5400 A for true temperatures from 1746° to 2785° K. A sodium photo-electric cell with an electrometer was employed for the measurements.

Previous determinations of the emissive power of tungsten have yielded results which are rather fragmentary and conflicting. Langmuir[1] has given a table which indicated that, for the visible region of the spectrum, ϵ increased as the wave-length decreased, and that ϵ increased slightly as the temperature increased. A. G. Worthing's[2] preliminary measurements showed that ϵ is greater for blue than for red light, and that ϵ unmistakably decreased with an increase in temperature. There are no data for the emissive power of tungsten in the ultra-violet.

The original plan of the present work had been to use a tungsten lamp equipped with a quartz window, and if possible to extend the measurements into that region of the ultra-violet where glass is opaque. A tungsten strip was mounted in a bulb to which was sealed normally a glass tube 1 cm in diameter and 3 cm in length. The end of the tube was closed by a quartz plate attached with red sealing-wax. Such a lamp could not be properly baked out during exhaustion, and so contained residual water-vapor. As a result, the tungsten deposited copiously on the walls of the bulb; no appreciable blackening occurred on the sides of the tube or on the quartz window. It was found that when the lamp was operated on a constant current, the temperature of the filament rose at the rate of many degrees per hour, owing to the wasting away of the metal, and a steady state was never reached. Spectrograms, Fig. 1, of a helical tungsten filament through a quartz window at various temperatures showed an appreciable amount of radiation in the ultra-violet as far as 2800 A. The temperatures assigned to each spectrogram of Fig. 1 are probably correct to within 50° K. A comparison of these spectrograms with that of a hydrogen vacuum tube, with which some experience had been had in respect to quantitative measurements in the ultra-violet, showed that there was no hope of obtaining any measurements on the tungsten radiation for wave-lengths below 3000 A, except by increasing the sensibility of the photo-electric apparatus many

[1] *Physical Review*, 7, 302, 1916. [2] *Ibid.*, 7, 497, 1916.

fold. Because of this, and because of the unsteadiness mentioned above, the plan of using the lamp with a quartz window was abandoned. All following measurements recorded in the present work have been made on the light from the tungsten in glass bulbs and have then been corrected for the transmission of the glass.

2. *Apparatus and experimental details.*—The tungsten was in the form of a strip, 32 mm$\times$2 mm, mounted vertically in a nitrogen atmosphere and inclosed in a glass bulb. Each lamp was carefully

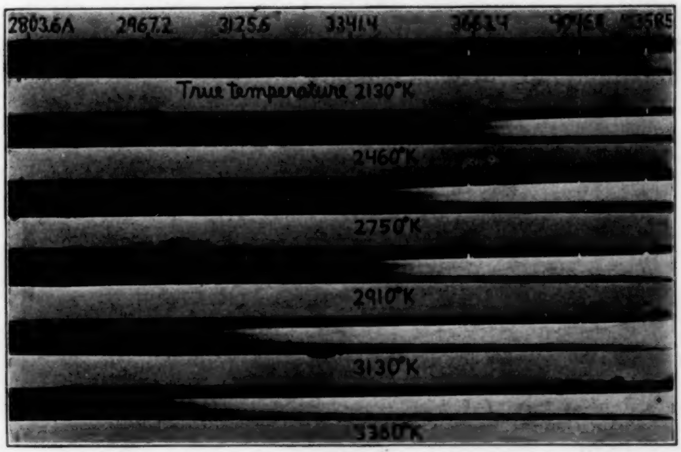


FIG. 1.—Short wave-length spectrum of tungsten at various temperatures for equal times of exposure with quartz-mercury lamp comparison spectrum (wave-lengths from R. A. Millikan, *Physical Review*, **7**, 364, 1916).

calibrated before the experiment, so that the relation between the current through the filament and the black-body temperature was known. The lamp was operated on storage-battery currents which were read on a calibrated ammeter, and the temperatures obtained from the calibration-curve were probably correct to 5° K. The true temperature was found from the black-body temperature by the use of Worthing's curve (*loc. cit.*).

The image of the tungsten strip T, Fig. 2, was focused by a quartz lens L, 2.8 cm in diameter and of focal length 15.8 cm, on the slit B of the spectrograph. Between the lamp and the quartz lens was an electrically operated shutter S controlled by a

pendulum with a period of five seconds. Close to the shutter was placed a rotating sector *R*, which enabled one to reduce the intensity of the light by a known fraction. The spectrograph consisted of the Rowland mounting of a concave speculum-metal grating *C* of 16 cm radius of curvature, ruled 15,000 lines to the inch, the area of ruling being 15 mm$\times$24 mm. The spectrum was brought to a focus at slit *A*, back of which was placed the sodium photo-electric cell *P* with a quartz window. Slit *A* and the cell were on a movable arm so that various wave-lengths could be reached. The dispersion was 106 A per mm. The cell was connected to a Dolezalek quadrant electrometer *E* of sensibility 1200 mm per volt of difference of potential between the quadrants (period 12 seconds).

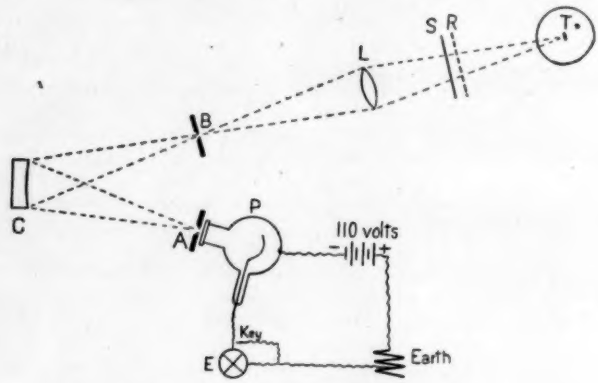


FIG. 2.—Diagram of apparatus

The steady-deflection method was employed. In this method the charge liberated in the cell during a known time of exposure to the light was collected on the insulated quadrants of the electrometer, and the deflection was taken after the needle had come to rest. The deflections recorded have in all cases been reduced to the unscreened deflection for an exposure of 10 seconds. For example, at wave-length 4326 A and temperature 2677° K., was recorded a deflection of 6930 mm. The deflection actually observed was 385 mm (the mean of 389, 383, 383, 385 mm), which was obtained by a 5^s exposure using a rotating sector which transmitted one-ninth of the light.

By means of the shutter, actuated by the pendulum, the times of exposure of the cell to the light were probably accurate to 0.5 per cent. The drift of the electrometer needle was negligible, being less than 1 mm per minute. In spite of these favorable conditions it was found impossible to repeat readings within an accuracy of 1 per cent; the deflections obtained under ostensibly identical experimental conditions often varied by as much as 4 per cent. The cause of this might have been due in part to fluctuations of the intensity of the light, and in part to unsteadiness in the cell. All the measurements recorded in this paper were the means of three, or more, observations, and were considered to be correct within 1 per cent for large deflections, the error being greater for the smaller deflections. The inability to repeat observations with precision was a serious defect of all the measurements. The most probable direction of improvement would have been in the use of a more sensitive cell, which would allow the sensibility of the electrometer to be decreased with a corresponding gain in steadiness. The cell used in the present work was one which had been prepared two years ago by the distillation of sodium *in vacuo*, and it seemed to have lost none of its sensitiveness. An attempt was made to use the cell in connection with a high resistance, and to employ the electrometer as a delicate galvanometer. It was found that this method was of a much lower order of sensitiveness than the steady-deflection method, and could not be used in the present work.

3. *The linear relation test of the photo-electric cell.*—The relation between the energy incident on the cell and the deflection of the electrometer was found to be a linear one for monochromatic radiation. The test was made in the following way: The transmissions of two neutral tint glasses for the blue mercury line at 4358 A were found to be 13.5 per cent and 13.9 per cent, respectively. The deflection due to the unscreened light was 2541 mm, to the light through the 13.5 per cent screen 343 mm, and to the light through the two screens 47.9 mm. The ratio of electrometer deflection to incident energy in each case was $\frac{2541}{1.00}$ or 2541, $\frac{343}{0.135}$ or 2541, and $\frac{47.9}{0.135\times0.139}$ or 2553, respectively. The ratio

was constant within the error of experiment, and hence the linear relation was considered satisfactorily established. Several sets of measurements were carried out with similar results.

4. *Scattered light.*—The spectrograph was inclosed in a light-tight metallic case blackened on the inside, and fitted with suitable diaphragms of smoked zinc. A visual examination with a transmission grating of the supposedly monochromatic beam of light issuing from slit A, when a tungsten lamp was used as a source, showed that the beam also contained light of all the other wave-lengths of the spectrum. This dilution of the beam with white light was due to the grating itself, which scattered the incident light in a manner similar to the effect produced by a rough surface. The intensity of this extraneous light was small, being about as strong as the light scattered by a camphor-smoked surface placed in front of the grating, and seemed to be roughly constant throughout the spectrum. When a mercury lamp was used as a source there appeared, in addition to general scattered light, at 5230 A a weak ghost of the green mercury line 5461 A. Data (Fig. 3) taken at the beginning of the work, and intended primarily for calibration purposes, have been used to obtain the corrections for scattered light. From curve 1, Fig. 3, the spectral photo-electric curve of a quartz-mercury lamp from 2000 A to 6000 A, it is seen that the deflections at dark portions of the spectrum, between the lines, dropped down to an approximately constant value, which indicated that the scattered light was fairly constant at all wave-lengths. On the strength of this the correction for scattered light has been taken to be the same for each wave-length. The intensity of the scattered light has been obtained from curve 2, Fig. 3. This curve gives the deflections taken throughout the spectrum of tungsten through glass at a constant temperature, from 2000 A to 6000 A. Below 3000 A the deflections were assumed to be due entirely to scattered light, and amounted to 30 mm at each wave-length, which was 1.1 per cent of the maximum deflection. The curve was then corrected by subtracting 30 mm from each deflection; this is shown in Fig. 3 by the dotted line drawn under curve 2. The corrections for the spectral photo-electric curves at the other temperatures were found by comparing their three greatest deflections with the corresponding deflections of curve 2,

Fig. 3; this amounted to a comparison of the areas under the curves with an accuracy sufficient for the purpose. In this way all the deflections have been corrected for the effect of scattered light.[1]

5. *The change in emissive power with wave-length.*—This was determined by measurements on a black body at the melting-point

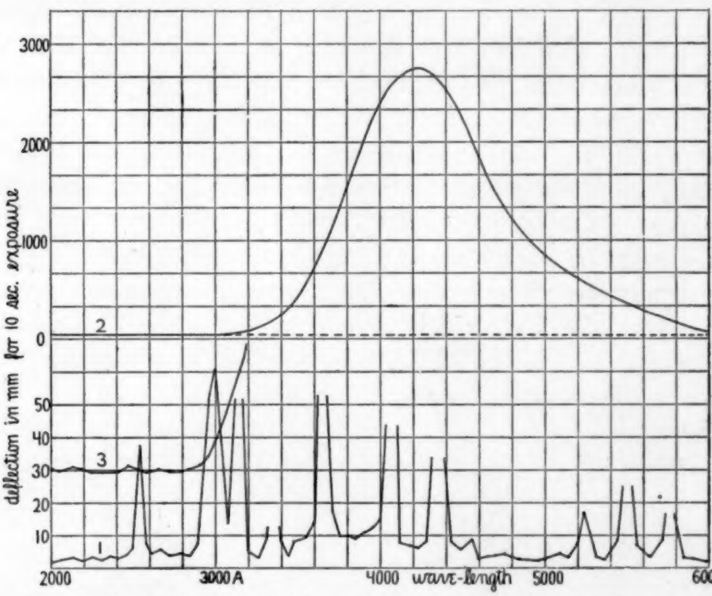


FIG. 3.—Spectral photo-electric curves. 1, quartz-mercury lamp; 2, tungsten lamp; 3, part of 2, enlarged. The dotted line under 2 shows the percentage attributed to scattered light.

of palladium, 1822°.5 K.,[2] and on the tungsten at 2143° K. Columns 1, 2, and 3, Table I, give the experimental data, corrected for scattered light. Column 4 gives the values of the "relative emissive power (uncorrected)" computed from the relation

$$\text{Relative } \epsilon \text{ (uncorrected)} = \frac{d_t}{d_b}\, e^{\frac{C_2}{\lambda}\left(\frac{1}{2143} - \frac{1}{1822.5}\right)}, \tag{2}$$

[1] An abstract of this work appeared in the *Journal of the Franklin Institute*, **182**, 695, 1916. At that time no corrections for scattered light were made. Therefore the conclusions stated there have been modified somewhat in the present paper. In the abstract, by mistake, the value for ϵ at 2143° for 4677 A was taken to be 0.470. The correct value is 0.475.

[2] A. L. Day and R. B. Sossman, *American Journal of Science*, **29**, 93, 1910.

where d_t and d_b are the deflections produced by the tungsten and black-body radiation, respectively, at wave-length λ, and C_2 is 14460 $\mu \times$ deg. K.[1] The values of column 4, already corrected for scattered light, were corrected for three other effects: (*a*) the finite width of slits, (*b*) the difference in the width of slit *A* in the two sets of measurements, (*c*) the loss of energy produced by the glass.

a) The corrections due to the finite width of slits, computed by the graphical method,[2] were negligible except for wave-lengths 3478 A and 5641 A, in which cases the correction amounted to an increase of 0.5 per cent in "relative ϵ (uncorrected)."

TABLE I

EMISSIVE POWER OF TUNGSTEN, AT TEMPERATURE 2143° K., AS FUNCTION OF THE WAVE-LENGTH

1	2	3	4	5	6
	ELECTROMETER DEFLECTION FOR 10 SEC. EXPOSURE CORRECTED FOR SCATTERED LIGHT				
WAVE-LENGTH	BLACK-BODY TEMP. 1822°K. SLIT A. 0.464 mm SLIT B. 0.502 mm	TUNGSTEN TEMP. 2143°K. SLIT A. 0.279 mm SLIT B. 0.502 mm			
	d_b Means of Four Series	d_t Means of Four Series	ϵ Relative (Uncorrected)	ϵ Relative (Corrected)	ϵ (Absolute Value)
3240 A......	0.2 mm	2.5 mm	....	....	....
3478........	4.1	23.9	0.191	0.533	0.485
3717........	11.6	79.7	.281	.543	.495
3956........	26.0	151.7	.289	.540	.493
4196........	44.0	223.1	.299	.552	.503
4435........	65.3	266.0	.279	.516	.470
4677........	70.6	252.7	.282	.521	.475
4916........	61.6	189.9	.275	.507	.463
5158........	41.0	110.4	.269	.498	.453
5400........	26.6	62.5	.260	.481	.438
5641........	21.0	42.4	.246	.457	.415
5890........	17.6	29.0	....	....	....

b) Because of the feebleness of the light from a black body at 1822.°5 K., slit *A*, Fig. 2, was opened wider for the measurements on the black body than for those on the tungsten. This produced an increase of 66.3 per cent in each value of column 4, Table I.

[1] E. P. Hyde, F. E. Cady, W. E. Forsythe, *Astrophysical Journal*, **42**, 294, 1915.

[2] E. P. Hyde, *ibid.*, **35**, 237, 1912.

c) In order to determine the transmission of the glass, the tungsten lamp was mounted between a quartz-mercury lamp and the spectrograph. The transmission of the two walls of the bulb was determined for certain mercury lines, and the transmission of one wall calculated. This was done for the glass at room temperature and for the glass when heated. The heating was effected by lighting the lamp and allowing the glass to become hot. Then the lamp was turned out and the transmission of the glass was measured immediately afterward. The data are shown in Fig. 4. The

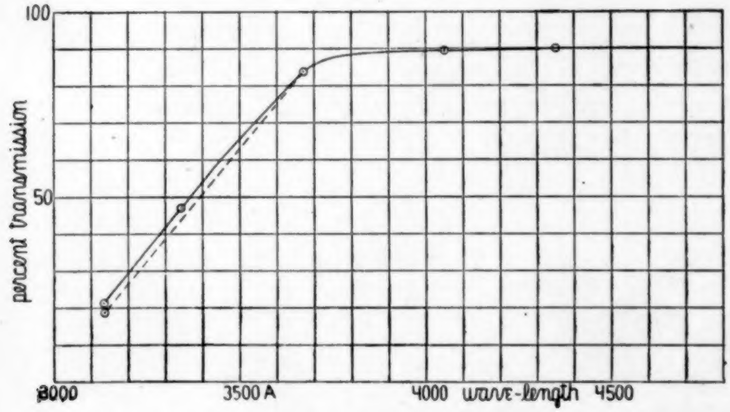
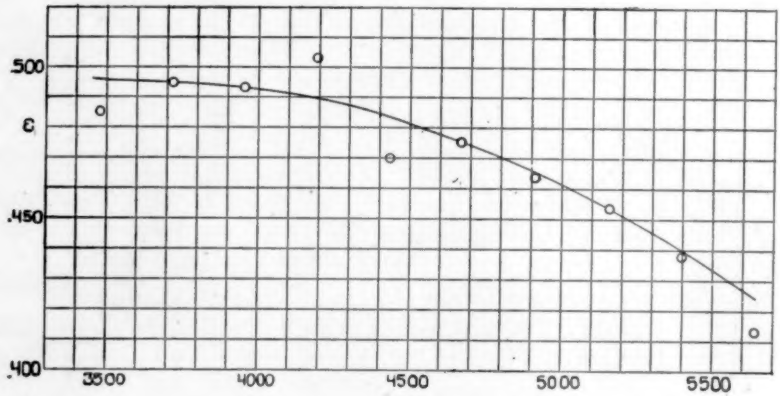


FIG. 4.—Transmission of glass of lamp bulb; full line, cold glass; dotted line, hot glass.

correction for the effect of the glass amounted to an increase in "relative ϵ (uncorrected)" of 66.6 per cent for 3478 A, 11.6 per cent for 3717 A, 11.3 per cent for 3956 A, and 11.1 per cent for all the other wave-lengths. The values of the emissive power thus corrected, "relative ϵ (corrected)," are given in column 5, Table I. The values of column 5 are 9 per cent greater than other observers have found in the regions of the spectrum where a comparison is possible. Several factors, the chief of which are (1) the difference in the width of slit *A* used in the two sets of measurements, (2) the existence of general scattered light, (3) the difficulty of lining up the apparatus so that the tungsten strip was accurately replaced by the black body, contribute to this discrepancy. Therefore the values in column 5 have been used only in determining the relative

changes in the emissive power with wave-length. In column 6 (see also Fig. 5) the absolute values of ϵ are given. These depend upon the choice of the value $\epsilon=0.475$ for wave-length 4677 A at 2143° K. (from unpublished measurements by A. G. Worthing).

6. *The change in the emissive power with temperature.*—The experimental procedure was as follows: with the photo-electric cell set for a certain wave-length the deflections were observed when the filament was operated at various temperatures. The logarithmic isochromatic curves, i.e., the curves formed by plotting the



Wave-lengths in A

FIG. 5—Emissive power of tungsten at temperature 2143° K.

logarithm of the deflection at constant wave-length against the reciprocal of the temperature, were obtained for a series of wave-lengths from 3295 A to 5532 A, for two lamps. The curves for lamp 2 are shown in Fig. 6; the curves for 4566, 4806, and 5047 A have been shifted arbitrarily along the X-axis to prevent them from intersecting. The deflections have been corrected for scattered light. The width of slit was 0.279 mm for slit A, and 0.502 mm for slit B; the correction for finite slit-width was negligible. The range of temperature was from 2103° K. to 2785° K. for all the lines, except for those for 4086, 4326, 4566, and 4806 A. In the case of these four it was possible to get readings for a temperature as low as 1746° K., because in this region of the spectrum of tungsten the

deflections were greatest. Fig. 6 shows that within the error of experiment the logarithmic isochromatic lines are rectilinear. Therefore, for each line

$$ln\ d = ln\ (J \times \text{const.}) = a - \frac{b}{T},$$

or

$$J = a'e^{-\frac{C_2'}{\lambda T}}, \tag{3}$$

d is the deflection produced at wave-length λ by J, the energy per unit wave-length, radiated per second per unit area by the tungsten at temperature T. Since the isochromatic lines are taken to

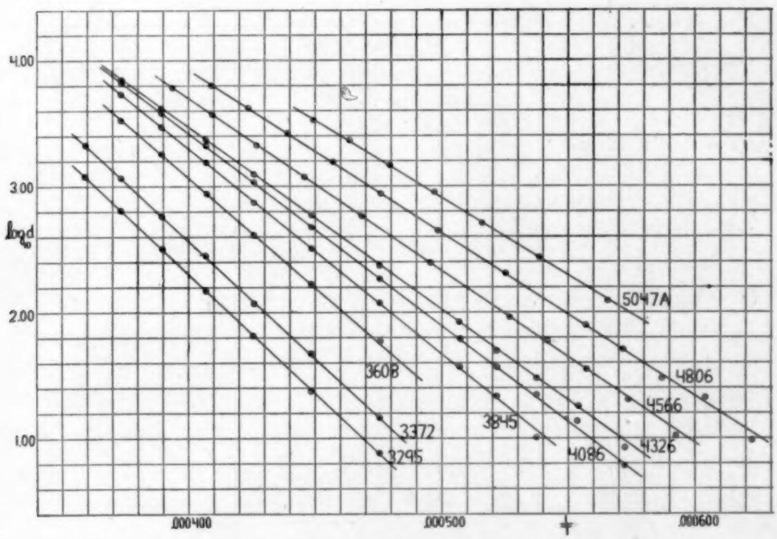


FIG. 6.—Logarithmic isochromatic curves of tungsten, lamp 2

be rectilinear for the range of temperatures under investigation, it follows that a' and C_2' are independent of T, but may be functions of λ. C_2' has been computed from the slope of each isochromatic line, and the value of C_2' for each wave-length is shown graphically in Fig. 7. The observed values of C_2' for 3295 A and 3372 A have been increased by 1.3 per cent and 0.3 per cent respectively, because of the temperature coefficient of transmission of the glass.

From equations (1) and (3) we obtain

$$ln\frac{\epsilon}{f(\lambda)}=\frac{C_2-C_2'}{\lambda T}. \tag{4}$$

Equation (4) has been used to compute the change in ϵ with T, using 14460 $\mu\times$deg. K. for C_2, and taking the values of C_2' for each wave-length from the straight line of Fig. 7.

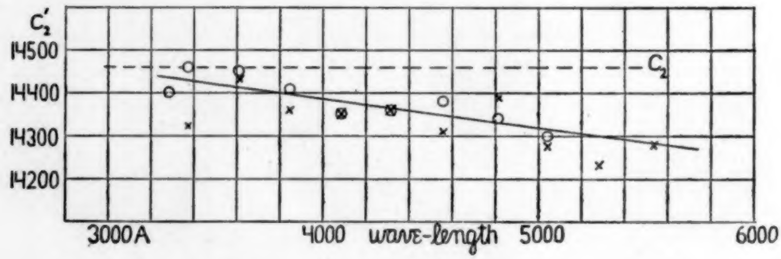


FIG. 7.—Values of C_2' in $\mu\times$ deg. K.
Crosses, lamp 1.
Circles, lamp 2.
Dotted line, $C_2=14460\mu\times$deg. K.
Values given by circles have been given greater weight.

7. *Emissive power of tungsten.*—The final values for the emissive power have been obtained by taking the values of ϵ at 2143° K. for each wave-length from the smooth curve of Fig. 5, and computing the values at the other temperatures by the use of equation (4) and the values at C_2' for each wave-length from Fig. 7. The results are shown graphically in Fig. 8. The change in ϵ with T, shown in Fig. 8, is in fair agreement with Worthing's measurements at 468 $\mu\mu$ and 665 $\mu\mu$.

It is worthy of mention that by means of Fig. 8, together with formula (1), the distribution of spectral energy of tungsten radiation can be computed. This enables one to use a tungsten lamp for the purpose of calibrating a spectro-photometric apparatus in energy terms.

8. *Accuracy of the results.*—The values of ϵ, given in Fig. 5, for those wave-lengths at the ends of the region of the spectrum under investigation were open to an error of perhaps 5 per cent, because in these cases the deflections were small and the corrections large.

The change in ϵ with T has been determined with greater precision; it is considered that given the value of ϵ for one temperature the data of this paper enable the computation of ϵ at another temperature with an error of about 1 per cent, for the range of temperatures and wave-lengths covered, the error being greater for the shorter wave-lengths.

9. *The effect upon the results of uncertainties in the values of C_2 and the melting-point of palladium.*—The values of ϵ given in Fig. 8

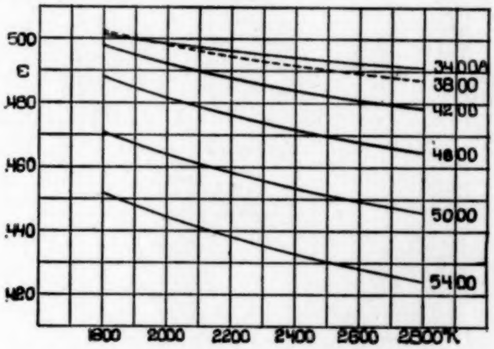


FIG. 8.—Emissive power of tungsten

have seemingly depended upon C_2 and the melting-point of palladium through the values of the temperatures used in the computations, and through the explicit occurrence of C_2 in equations (2) and (4). It is shown, however, in the following paragraphs that the values of ϵ are independent of any knowledge of the values of the two constants, C_2 and the melting point of palladium.

The true temperature, T°K., of the tungsten has been found by the use of Worthing's scale from the known black-body temperature S°K. T, on Worthing's scale, is defined by the equation

$$\frac{1}{T}=\frac{1}{S}-\frac{\lambda_r \ln \epsilon_r}{C_2},$$

where ϵ_r is the emissive power for wave-length λ_r, and the temperature S, as measured by Worthing (*loc. cit.*).

S is defined by the equation

$$\frac{1}{S}=\frac{1}{P}-\frac{\lambda_r \ln R}{C_2},$$

where $P°$K. is the melting-point of palladium, and R is the ratio of the brightness of a black body at T to its brightness at P, measured at wave-length λ_r.

Combining these two equations

$$\frac{1}{T}=\frac{1}{P}-\frac{\lambda_r \, ln\epsilon_r R}{C_2},$$

or

$$\frac{1}{T}=\frac{1}{P}-\frac{a}{C_2}, \tag{5}$$

where a is a quantity independent of C_2 and P.

Consider the change in ϵ with λ as given by (2). Writing (2) in a more general form, by putting T for 2143° K. and P for 1822°5 K., gives

$$\epsilon=\frac{d_t}{d_b}\, e^{\frac{C_2}{\lambda}\left(\frac{1}{T}-\frac{1}{P}\right)}.$$

Substituting for $\frac{1}{T}$ from (5),

$$\epsilon=\frac{d_t}{d_b}\, e^{-\frac{a}{\lambda}}$$

which is free from C_2 and P.

Consider the change in ϵ with T as given by (4). C_2' in (4) may be considered to be computed from the co-ordinates of two points, $\left(ln\ d_1,\ \frac{1}{T_1}\right)$ $\left(\text{and } ln\ d_2,\ \frac{1}{T_2}\right)$, which lie on the logarithmic isochromatic line for wave-length λ, by the relation

$$C_2'=\frac{\lambda\ ln\frac{d_2}{d_1}}{\frac{1}{T_1}-\frac{1}{T_2}},$$

where d_1 and d_2 are the deflections at T_1 and T_2, respectively. Substituting for $\frac{1}{T_1}$ and $\frac{1}{T_2}$ from (5),

$$C_2'=C_2\frac{\lambda\ ln\frac{d_2}{d_1}}{a_2-a_1}.$$

a_1 and a_2, given by (5), correspond to T_1 an T_2, respectively. Using (4), we may write the relation between the values of the emissive powers ϵ_1 and ϵ_2 for temperatures T_1 and T_2, respectively, at wave-length λ:

$$ln\frac{\epsilon_1}{\epsilon_2}=\frac{C_2-C'_1}{\lambda}\left(\frac{1}{T_1}-\frac{1}{T_2}\right).$$

Substituting for C'_2, $\frac{1}{T_1}$ and $\frac{1}{T_2}$, this becomes

$$ln\frac{\epsilon_1}{\epsilon_2}=\frac{a_2-a_1}{\lambda}-ln\frac{d_2}{d_1},$$

which is free from C_2 and P.

Therefore the present method of determining the changes in the emissive power is independent of a knowledge of the values of C_2 and the melting-point of palladium.

10. *Summary.*—The radiation from tungsten has been studied by means of a sodium photo-electric cell and electrometer throughout the region of the spectrum from 3400 to 5400 A, for true temperatures, on Worthing's scale (*loc. cit.*), from 1746° to 2785° K. From the measurements the values for the emissive power of tungsten have been computed; these values are independent of a knowledge of the radiation constants C_2 and the melting-point of palladium.

In conclusion, it is a pleasure to acknowlege my indebtedness to the director and members of the staff of the Nela Research Laboratory. Dr. W. E. Forsythe has calibrated lamps and manipulated the black body for me. I desire to express especially my appreciation of the many helpful suggestions and kindly criticism of Dr. A. G. Worthing. The work was carried out under the Charles F. Brush Summer Fellowship in Physics.

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STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

By HARLOW SHAPLEY

FOURTH PAPER: THE GALACTIC CLUSTER MESSIER 11[1]

Of the many striking stellar clusters in the great galactic clouds extending through Ophiuchus, Sagittarius, and Scorpio, none has received as much attention as Messier 11,[2] the very rich open aggregation of stars in the small constellation of Scutum Sobieski. Discovered by Kirch two hundred and thirty-five years ago, it has since been observed and described by nearly all students of clusters, nebulae, and comets.[3] In 1836–1839 a catalogue of micrometrically determined positions of 184 of its stars was undertaken by Lamont at Munich.[4] Thirty years later Helmert at Hamburg repeated the measures, forming a catalogue of 200 stars and comparing his results with those of the earlier astronomer.[5] The favorable length of the interval since the work of Lamont and the serviceable check by Helmert's later catalogue led Stratonoff at Tachkent in Turkestan to redetermine in 1896 with the more accurate methods then available the relative position of the stars, and to deduce proper motions for the cluster as a whole and for its individual members.[6] To accomplish this the positions were measured on photographic plates, not only for the stars in the cluster, but for several hundred stars outside its bounds. The 861 objects, whose co-ordinates to the hundredth of a second of arc are tabulated by Stratonoff, include

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 126.

[2] Messier 11 = h.2019 = G.C.4437 = N.G.C.6705; position for 1900 (from *Harvard Annals*, **60**, 214, 1908), $\alpha = 18^h\ 45^m.7$, $\delta = -6°23'$; galactic longitude = 355°, galactic latitude = −3°. The conspicuous bright star in the south-following edge of the cluster is Lalande 35043 = B.D. −6°4929, etc.

[3] For instance, see the paper by Barnard in *Monthly Notices*, **57**, 15, 1896, and the descriptive and historical notes by Miss Clerke, *The System of the Stars*, p. 229 (London, 1905).

[4] *Annalen der königlichen Sternwarte bei München*, **17**, 305.

[5] *Publicationen der Hamburger Sternwarte*, No. 1, 1874.

[6] *Publications de l'Observatoire astronomique et physique de Tachkent*, No. 1, 1899.

practically all the stars in an area of 700 square minutes which are brighter than a certain magnitude (near the limit of his plates). That the comparison of his positions with those of sixty years earlier gave only negative results for proper motion does not detract from the value of the work, for his result indicates the great distance of the stars involved, and the new measures of position afford a much sounder basis for the future investigation of motions. Stratonoff also determined the photographic magnitudes; but since his values are based on estimates of brightness taken from the *B.D.* catalogue, no especial accuracy is claimed for them.

The present study of colors and magnitudes in Messier 11 was undertaken chiefly because of the cluster's low galactic latitude and its position with reference to the star clouds of the southern Milky Way. The open cluster discussed in the preceding paper of this series,[1] Messier 67, is situated in galactic latitude $+34°$. Its constituency differs little from the non-cluster stars of that neighborhood, and the conclusion was reached that, unlike the globular clusters, it is nothing more than a condensation in an ordinary stellar field. The faint stars in that locality are of the redder color-classes, in agreement with Seares's result for the North Polar stars,[2] galactic latitude $+28°$. A different result might be anticipated for stars in Messier 11 and for the fainter members of the rich galactic cloud which forms its background. The determination of color-classes in such regions thus contributes directly to the problem of the extent of the Milky Way, and bears as well on the relation of open stellar groups to the surrounding clouds of stars.[3]

The large angular distance of Messier 11 from the reference field of standard magnitudes at the North Pole has made it difficult to secure good intercomparison photographs, and it has been necessary to reject many plates without measurement, either because of deformed images, or because of marked differences in the seeing in the directions of the cluster field and the North Pole. The plates

[1] *Mt. Wilson Contr.*, No. 117, 1916.

[2] *Ibid.*, No. 81; *Astrophysical Journal*, **39**, 361, 1914.

[3] Excellent photographs by Barnard of the star clouds in the region of Messier 11 are published in *Lick Observatory Publications*, **11**, 1915.

finally adopted and used for the determination of magnitudes are described in Table I. The time and hour-angle for each plate refer to the middle of the exposure on the cluster.

TABLE I

LIST OF PLATES

Plate Number	Date, G.M.T.	Kind of Plate	Number of Exposures	Exposure Time	Hour-Angle	Extinction Correction
2467.....	1915, June 7.965	Iso	3	10^m	23°5 W	+0ᴹ06
2468.....	7.960	Seed 27	3	2	21.5 W	+0.11
2638.....	Aug. 16.804	Iso	3	5	34 W	+0.02
2639.....	16.803	Seed 27	3	2	33 W	+0.06
3180.....	1916, July 5.710	Iso	2	10	40 E	0.00
3181.....	5.704	Seed 27	3	1	42 E	−0.02
3235.....	8.753	Iso	3	15	22 E	+0.06
3236.....	8.746	Seed 27	3	3*	25 E	+0.11

* Duration of exposure on cluster uncertain; the measures on Polar Standards rejected and scale of plate and zero point determined by comparisons with results from other Seed 27 plates.

The method of measurement and reduction is sufficiently indicated in the preceding papers.[1] In order to abbreviate the catalogue of magnitudes, the residuals are omitted and a brief discussion of them is given in Tables II and III. An error in the timing of one exposure on Plate 3236 (not discovered until the plates were measured) made it necessary to determine the scale from the results of the other Seed 27 plates. To do this the mean magnitudes in the eighth column of Table II were plotted against the scale-readings for the corresponding stars on Plate 3236, and the resulting curve was employed to determine magnitudes for all stars measured on that plate.

The residuals for the different plates in Tables II and III are the deviations (in hundredths of a magnitude) from the tabulated means after the systematic deviations at the bottom of the table have been applied. The stars included in these two tables are chiefly near the center and are the most difficult in the cluster. The average errors for the more distant stars are appreciably smaller. The tabulated residuals show that no important errors are present. For one plate, which showed diversely irregular images for the more

[1] *Mt. Wilson Contr.*, Nos. 115, 116, 117.

TABLE II

RESIDUALS FOR PHOTOGRAPHIC MAGNITUDES

Star	Plate 2468		Plate 2639		Plate 3181		Mean of Three Plates	Plate 3236		Mean of All Plates*
	Mag.	Res.	Mag.	Res.	Mag.	Res.		Mag.	Res.	
449	14.37	− 4	14.18	−10	14.21	+ 1	14.25	14.43	+13	14.30
452	......	......	12.71	+12	12.46	− 5	......	12.66	+ 5	12.61
459	14.27	+12	13.88	−14	13.79	−15	13.98	14.21	+17	14.04
462	12.84	− 6	12.79	+ 2	12.63	− 6	12.75	12.90	+11	12.79
466	13.90	− 7	13.77	− 7	......	......	......	13.90	+ 4	13.86
466*a*	14.89	− 7	14.87	+ 4	......	......	......	14.78	− 7	14.85
472	13.06	+10	12.84	+ 1	12.71	− 4	12.87	12.80	− 5	12.85
472*a*	15.74	+ 3	15.67	+ 9	15.39	−11	15.60	15.60	0	15.60
476	13.09	−18	13.15	+ 1	13.25	+19	13.16	13.14	− 2	13.16
491	12.37	− 8	12.38	+ 6	......	......	......	12.28	− 6	12.34
501	12.07	0	12.11	+17	11.71	−15	11.96	11.95	− 1	11.96
503	14.76	+13	14.40	−10	14.48	+ 6	14.55	14.44	− 8	14.52
516	12.20	− 6	12.12	− 1	......	......	......	12.13	− 2	12.15
517	13.73	+12	13.48	0	13.29	−11	13.50	13.49	− 1	13.50
520	15.13	− 8	14.96	−12	15.03	+ 3	15.04	15.28	+18	15.10
520*a*	15.40	−13	15.53	+13	15.39	+ 7	15.44	15.38	− 4	15.42
523	12.44	+ 8	12.30	+ 7	12.02	−13	12.25	12.23	− 2	12.25
523*a*	15.50	0	15.30	− 7	15.33	+ 4	15.38	15.43	+ 4	15.39
524	14.13	− 2	13.92	−10	14.00	+ 6	14.02	14.11	+ 7	14.04
533	13.57	+ 1	......	......	13.24	−11	......	13.53	+ 8	13.45
533′	14.43	+10	14.11	− 9	14.16	+ 4	14.23	14.16	− 6	14.22
534	12.67	−14	12.82	+14	......	......	......	12.61	− 9	12.70
538	......	......	12.62	+ 8	12.50	+ 4	......	12.55	− 1	12.56
538*a*	14.60	+10	14.23	−14	14.23	− 6	14.35	14.48	+ 9	14.39
539	13.30	−11	......	......	13.10	−10	......	13.49	+19	13.30
542	14.23	+ 8	13.84	−18	......	......	......	14.04	0	14.04
542*a*	15.00	+13	14.51	−23	......	......	......	14.78	+ 2	14.76
543	15.19	−22	15.49	+21	15.23	+ 3	15.30	......	......	15.30
544	12.07	− 1	......	......	11.71	−16	......	12.13	+16	11.97
551	12.77	+ 5	12.59	0	12.46	− 5	12.61	12.63	+ 2	12.61
553	15.09	+ 4	14.91	− 1	14.91	+ 7	14.97	14.84	−10	14.94
554	14.06	+ 1	......	......	13.90	+ 6	......	13.85	− 9	13.94
556	13.90	+13	13.66	− 2	13.54	− 2	13.70	13.53	−13	13.66
558	14.56	+ 3	......	......	14.41	+ 9	......	14.29	−13	14.42
562	14.53	−14	......	......	14.65	+19	......	14.49	− 7	14.56
565	13.84	+ 7	13.68	+ 4	13.54	− 2	13.69	13.57	− 9	13.66
570*a*	15.65	+16	15.41	+ 5	15.22	− 6	15.43	15.23	−15	15.38
572	13.47	+ 4	13.36	+ 6	13.14	− 8	13.32	13.33	+ 1	13.32
572*a*	15.43	−24	15.67	+13	15.51	+ 5	15.54	15.64	+ 8	15.56
581	11.90	− 5	......	......	11.72	− 2	......	11.91	+ 7	11.84
582	......	......	14.26	− 3	14.29	+ 8	......	14.39	+ 8	14.31
586	13.93	+10	13.68	− 2	13.58	− 4	13.73	13.71	− 1	13.72
589	15.22	0	15.19	+10	15.06	+ 5	15.16	14.95	−16	15.11
593	13.90	− 9	13.95	+ 9	13.72	− 6	13.86	13.93	+ 5	13.88
593*a*	15.53	− 5	15.49	+ 4	15.41	+ 4	15.48	15.43	− 4	15.47
600	14.43	+13	14.04	−13	14.21	+12	14.23	14.08	−11	14.19
600*a*	15.97	−10	15.91	− 3	16.16	+30	16.01	15.81	−15	15.96
604*a*	15.87	+ 8	15.59	− 7	15.49	− 9	15.65	15.76	+ 8	15.68
607	13.64	+ 9	13.41	− 1	13.26	− 8	13.44	......	......	13.44
622	14.37	−13	......	......	14.56	+27	......	14.25	−14	14.39
No. of values	47		42		43			48		
Av. dev	±0.085		±0.079		±0.085			±0.076		
Syst. dev	+0.11		−0.02		−0.10			....		

* In some cases these means differ slightly from the final mean photographic magnitudes in the catalogue, because of the application of the systematic corrections derived with the aid of this table.

distant stars, only images near the center were measured; and for Plate 3235 the relatively long exposure of 15 minutes made impossible the accurate measurement of the brighter stars.

TABLE III

RESIDUALS FOR PHOTO-VISUAL MAGNITUDES

Star	Plate 2467		Plate 2638		Plate 3180		Plate 3235		Mean Pv. Mag.	Color-Index
	Mag.	Res.	Mag.	Res.	Mag.	Res.	Mag.	Res.		
449	14.19	+ 1	14.16	+ 2	14.26	+ 2	14.20	− 4	14.20	+0.10
457	13.18	−10	13.36	+12	13.25	− 9	13.40	+ 6	13.30	+0.36
459	13.77	+ 1	13.56	−16	13.80	− 2	14.00	+18	13.78	+0.26
462a	14.66	+ 6	14.53	− 3	14.63	− 3	14.66	0	14.62	+0.39
472a	14.64	+ 3	14.57	0	14.73	+ 6	14.57	−10	14.63	+0.97
474	14.04	+12	14.17	+19	14.00	− 8	13.96	−12	14.04	+0.40
481	13.68	− 8	13.71	− 1	13.84	+ 2	13.89	+ 7	13.78	+0.10
485	13.97	−11	14.14	+10	14.07	− 7	14.21	+ 7	14.10	+0.44
492	13.72	− 2	13.76	+ 6	13.80	0	13.74	− 6	13.76	+0.34
502	13.97	− 6	13.92	− 7	14.10	+ 1	14.21	+12	14.05	+0.19
503	14.24	+ 5	14.23	+ 8	14.33	+ 8	14.04	−21	14.21	+0.31
513	14.22	+ 2	14.05	−11	14.29	+ 3	14.32	+ 6	14.22	+0.24
514	14.76	+ 5	14.61	− 6	14.83	+ 6	14.72	− 5	14.73	+0.40
520	14.47	+ 6	14.29	− 8	14.60	+13	14.35	−12	14.43	+0.67
520a	14.84	− 2	14.88	+ 8	15.04	+14	14.69	−21	14.86	+0.57
522	12.96	− 6	12.99	+ 1	13.03	− 5	13.18	+10	13.04	+0.57
523a	14.96	+ 5	14.79	− 8	14.91	− 6	15.06	+ 9	14.93	+0.46
524	14.00	+ 8	13.77	−11	13.95	− 3	14.04	+ 6	13.94	+0.10
532	14.06	+ 1	14.11	+10	14.10	− 1	14.00	−11	14.07	+0.03
533′	14.13	+ 3	14.03	− 3	14.13	− 3	14.20	+ 4	14.12	+0.10
538a	14.17	−14	14.31	+ 4	14.42	+ 5	14.42	+ 5	14.33	+0.06
543	14.90	+ 6	14.76	− 4	14.84	− 6	14.93	+ 3	14.86	+0.34
553	14.76	+11	14.53	− 8	14.63	− 8	14.77	+ 6	14.67	+0.27
452	12.08	+21	11.70	−13	11.89	− 8	.....	.....	11.89	+0.76
461	12.74	+ 1	12.65	− 4	12.86	+ 3	.....	.....	12.75	+0.43
462	12.23	+ 1	12.18	0	12.32	0	.....	.....	12.24	+0.55
472	12.31	− 1	12.33	+ 5	12.39	−3	.....	.....	12.34	+0.52
476	12.34	0	12.33	+ 3	12.42	− 2	.....	.....	12.36	+0.80
477	12.65	− 3	12.72	+ 8	12.72	− 6	.....	.....	12.70	+0.54
482	13.04	0	12.99	− 1	13.16	+ 2	.....	.....	13.06	+0.43
486	12.31	−12	12.44	+ 5	12.59	+ 6	.....	.....	12.45	+0.40
491	11.21	−10	11.41	+14	11.36	− 5	.....	.....	11.33	+1.00
493	12.01	+11	11.76	−10	12.00	0	.....	.....	11.92	+0.63
498	12.71	+ 2	12.59	− 6	12.83	+ 4	.....	.....	12.71	+0.50
516	11.42	− 6	11.65	+21	11.42	−16	.....	.....	11.50	+0.62
523	11.45	− 6	11.33	−14	11.81	+20	.....	.....	11.53	+0.72
534	11.26	+ 1	11.16	− 5	11.40	+ 5	.....	.....	11.27	+1.40
538	12.05	+ 2	11.98	− 1	12.11	− 2	.....	.....	12.05	+0.55
551	12.11	+ 3	12.05	+ 1	12.15	− 3	.....	.....	12.10	+0.52
No. of values	39		39		39		23			
Syst. dev.										
>13 Mag.	−0.02		−0.06		+0.04		+0.04			
<13 Mag.	−0.02		−0.06		+0.08		....			
Av. dev.	±0.052		±0.071		±0.053		±0.087			

With a few exceptions, all the magnitudes of the main catalogue (Table IV) depend upon at least two plates. The numbers in the first column are those assigned by Stratonoff. Stars marked with an asterisk are omitted from the statistical discussion, either because neighboring images are so close that the Eberhard effect may have influenced their magnitudes, or because of uncertainty in the measures. Except for close groups and double stars, the general concentration of images on the plates used is not sufficient to cause fear of perceptible errors from the Eberhard effect or similar phenomena. The residuals give no evidence of a differential effect of this kind between the plates; and, even for stars nearly in contact, whatever errors enter affect mainly magnitudes and not colors, for the densities of the stellar images on the two kinds of plates are closely comparable. In choosing stars for statistical discussion, however, liberal exclusion was made wherever possibility of such systematic errors was suspected. The results tabulated in Tables VI and VII should, therefore, be completely free from systematic or accidental errors depending on stellar condensation.

The numbers with postscript letters in Tables II, III, and IV refer to objects not listed by Stratonoff—those with appended Greek letters indicating the components of double or triple stars, and those with italic letters referring either to close faint companions or merely to neighboring uncatalogued stars of a photovisual brightness comparable with that of the stars catalogued. Using Stratonoff's stars for reference points, identification positions of the postscript stars were determined by measurements with a millimeter scale on enlarged positives. The right ascensions and declinations so obtained are listed in Table V. For positions of the Stratonoff stars reference must be made to the Tachkent catalogue.

As a matter of convenience in discussing colors and magnitudes, the cluster was divided into several concentric regions. The numbers in the second column of Table IV refer to these divisions, Region 1 including all stars within $0\overset{'}{.}5$ from the center, Region 2 including all stars between $0\overset{'}{.}5$ and $1\overset{'}{.}0$, Region 3 those between $1\overset{'}{.}0$ and $1\overset{'}{.}5$, etc. The star at the point adopted as center of the cluster is No. 462, for which the position for 1900 is:

$$\text{R.A.} = 18^{h}\ 45^{m}\ 42\overset{s}{.}75, \qquad \text{Dec.} = -6^{\circ}\ 23'\ 12\overset{''}{.}8.$$

TABLE IV

PHOTOMETRIC CATALOGUE OF 458 STARS IN MESSIER 11

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
204...	7	13.38	13.12	+0.26	253..	5	14.50	14.31	+0.19
205...	9	13.86	13.76	+0.10	257..	6	15.26	14.84	+0.42
206...	7	14.50	14.34	+0.16	258*.	5	14.24	13.86	+0.38
207...	8	12.75	12.24	+0.51	258a.	5	15.80	15.00	+0.80
208...	7	14.39	13.98	+0.41	260*.	6	14.06	13.72	+0.34
210...	8	14.76	14.58	+0.18	261*.	7	14.40	14.19	+0.21
211...	7	12.26	11.62	+0.64	262..	7	13.54	13.31	+0.23
212...	9	13.68	13.61	+0.07	263..	5	14.82	14.69	+0.13
213...	7	14.37	14.08	+0.29	264..	5	14.82	14.62	+0.20
214...	7	13.48	13.18	+0.30	265*.	7	13.34	13.32	+0.02
217...	8	14.70	14.54	+0.16	266*.	6	13.89	13.53	+0.36
218...	8	14.42	14.20	+0.22	266a.	6	15.24	14.82	+0.42
219...	6	14.64	14.54	+0.10	266b.	6	15.17	13.44	+1.73
221...	6	13.07	12.86	+0.21	267..	5	13.26	12.78	+0.48
224...	6	13.56	13.58	−0.02	267a.	5	14.76	13.30	+1.46
225...	6	13.78	13.64	+0.14	267b*	5	15.20	14.70	+0.50
226*..	8	14.09	13.76	+0.33	268..	6	14.46	14.25	+0.21
226a..	7	14.40	14.50	−0.10	272..	5	13.76	13.36	+0.40
226b..	7	14.90	13.46	+1.44	273..	4	13.00	12.54	+0.46
226c..	8	15.38	13.50	+1.88	273a.	4	15.41	14.91	+0.50
227...	6	12.06	11.31	+0.75	274..	5	14.32	14.12	+0.20
229...	6	13.57	13.48	+0.09	275..	7	14.86	14.72	+0.14
230...	6	14.82	14.71	+0.11	276..	7	14.43	14.26	+0.17
232...	6	12.22	11.89	+0.33	277..	8	14.54	14.28	+0.26
233α*.	6	14.52	12.40	....	278..	4	14.32	14.11	+0.21
233β*.	6	14.68		....	280..	7	13.01	11.92	+1.09
234*..	8	14.57	14.36	+0.21	283..	4	14.08	13.92	+0.16
236...	6	13.07	12.61	+0.46	284..	5	14.49	14.26	+0.23
237...	7	11.74	11.02	+0.72	285..	4	13.08	12.38	+0.70
238...	8	14.58	14.32	+0.26	287..	4	14.56	14.28	+0.28
239...	5	13.24	13.13	+0.11	287a.	4	15.23	14.69	+0.54
240...	5	13.21	11.81	+1.40	288..	4	12.75	12.13	+0.62
242...	5	14.00	14.16	−0.16	291..	6	14.82	14.50	+0.32
243...	8	14.07	13.92	+0.15	292*.	4	13.22	12.80	+0.42
245...	5	12.15	11.89	+0.26	293*.	4	12.56	11.71	+0.85
247...	7	13.12	11.07	+2.05	293a*	5	15.29	14.76	+0.53
248*..	5	14.64	14.48:	+0.16:	296..	5	13.89	13.64	+0.25
248a*.	5	15.50	14.17:	+1.33:	296a.	5	14.86	14.70	+0.16
249...	6	13.92	13.80	+0.12	297..	4	12.44	11.68	+0.76
249a..	5	15.28	14.82	+0.46	298..	4	13.91	13.63	+0.28
249b..	5	15.14	14.56	+0.58	300..	6	12.28	11.86	+0.42
250...	5	13.08	12.60	+0.48	301..	6	15.16	14.64	+0.52
251...	6	13.34	13.12	+0.22	302..	4	15.24	14.77	+0.47
251a..	5	15.11	14.82	+0.29	303..	4	14.69	14.55	+0.14
252...	7	13.08	11.12	+1.96	307..	5	14.20	14.02	+0.18

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
308*..	4	14.96	14.65	+0.31	358..	3	13.84	13.66	+0.18
310...	4	13.04	12.52	+0.52	359..	3	14.33	14.25	+0.08
310a..	3	15.60	14.34	+1.26	360*.	4	14.62	14.25	+0.37
314...	5	14.28	14.16	+0.12	362..	5	13.68	13.28	+0.40
315...	4	14.48	14.19	+0.29	363..	3	14.00	13.86	+0.14
317*..	6	14.36:	14.14	+0.22:	364..	4	12.73	11.44	+1.29
320...	5	14.84	14.67	+0.17	365*.	3	13.32	12.55	+0.77
321...	6	14.86	14.56	+0.30	366..	3	13.02	12.52	+0.50
322*..	6	13.46:	13.12	+0.34:	368..	3	14.08	13.60	+0.48
323*..	4	13.54	12.82	+0.72	369*.	3	14.44	13.76	+0.68
323a*.	4	14.66	14.51	+0.15	370*.	3	15.04	14.64	+0.40
324...	4	13.80	13.70	+0.10	372*.	3	14.56	14.10	+0.46
324a..	4	15.07	14.76	+0.31	373..	2	14.86	14.68	+0.18
325...	3	13.04	12.48	+0.56	374..	2	14.46	14.10	+0.36
328...	3	14.11	13.84	+0.27	375..	3	12.54	11.97	+0.57
329...	3	14.65	14.30	+0.35	376..	4	14.10	13.58	+0.52
330...	4	14.66	14.26	+0.40	376a.	4	15.11	14.78	+0.33
330a..	4	15.84	14.68	+1.16	377..	4	13.64	13.27	+0.37
331...	4	12.24	11.14	+1.10	378..	2	14.18	13.87	+0.31
333...	6	14.06	13.86	+0.20	379..	4	14.76	14.31	+0.45
334...	3	14.18	13.95	+0.23	380..	3	14.49	14.16	+0.33
335...	3	15.24	14.84	+0.40	381..	4	12.44	11.81	+0.63
336...	3	12.08	11.29	+0.79	381a.	4	15.29	14.27	+1.02
336a..	3	15.26	14.66	+0.60	383..	4	14.12	14.04	+0.08
337...	4	14.61	14.24	+0.37	384..	6	14.58	14.52	+0.06
338...	4	13.36	12.90	+0.46	385..	5	12.96	12.18	+0.78
338a..	5	15.60	15.00	+0.60	386..	5	14.64	14.37	+0.27
341...	3	12.42	11.78	+0.64	387..	3	12.84	11.30	+1.54
342...	3	13.22	11.47	+1.75	389..	2	14.67	14.36	+0.31
343...	4	14.18	13.74	+0.44	391..	2	13.18	12.76	+0.42
344...	5	13.23	12.62	+0.61	392..	3	15.04	14.74	+0.30
345...	4	12.06	11.32	+0.74	393..	3	13.54	13.18	+0.36
345a..	4	15.30	14.80	+0.50	394..	2	14.16	13.92	+0.24
346...	3	12.82	12.46	+0.36	394a.	2	15.34	15.00	+0.34
347...	4	13.32	11.74	+1.58	395*.	5	12.19	11.31	+0.84
348...	4	14.84	14.60	+0.24	396..	3	15.16	14.92	+0.24
348a..	4	15.13	14.62	+0.51	397..	4	14.98	14.62	+0.36
350...	6	14.42	14.40	+0.02	397a.	5	15.68	15.02	+0.66
351...	5	13.45	13.06	+0.39	398..	2	13.84	13.51	+0.33
352...	5	13.69	13.32	+0.37	400*.	3	14.68	14.28	+0.40
353...	3	13.68	13.31	+0.37	402..	4	13.06	12.53	+0.53
354...	6	14.55	14.52	+0.03	403..	3	13.53	13.24	+0.29
355...	3	14.49	14.04	+0.45	404..	3	14.36	14.02	+0.34
356*..	4	12.60	11.94	+0.66	405..	7	13.18	12.93	+0.25
357...	5	14.78	14.52	+0.26	406..	6	14.99	14.70	+0.29

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
407...	3	11.93	11.02	+0.91	456..	3	12.30	11.44	+0.86
408...	3	13.00	12.52	+0.48	456a.	3	15.01	14.64	+0.37
409...	2	13.74	13.11	+0.63	457*.	1	13.66	13.30	+0.36
410*..	5	14.80	14.18	+0.62	458..	2	14.09	13.84	+0.25
411...	1	14.04	13.84	+0.20	459..	1	14.04	13.78	+0.26
411a..	2	15.42	15.10	+0.32	460..	2	12.97	12.63	+0.34
412...	2	13.54	12.18	+1.36	460a.	3	15.50	14.90	+0.60
414*..	3	13.96	13.54	+0.42	461..	2	13.18	12.75	+0.43
417*..	5	14.64	14.13	+0.51	461a.	2	15.59	14.90	+0.69
419...	3	15.36	14.80	+0.56	462..	1	12.79	12.24	+0.55
419a..	4	15.00	14.72	+0.28	462a.	1	15.01	14.62	+0.39
420...	2	13.42	13.28	+0.14	464..	3	13.94	13.74	+0.20
420a..	3	15.43	14.99	+0.44	465..	3	13.12	12.62	+0.50
421...	3	13.13	12.60	+0.53	466*.	1	13.82	13.34	+0.48
421a*.	3	15.42	14.56:	+0.86:	466a*	1	14.82	13.40	+1.42
422*..	3	13.22	12.66	+0.56	467..	4	15.14	14.64	+0.50
423...	1	13.46	11.83	+1.63	468..	4	13.91	13.62	+0.29
424...	6	12.72	12.23	+0.49	468a.	4	15.10	14.67	+0.43
425...	2	12.62	12.12	+0.50	469..	2	13.78	13.42	+0.36
426*..	3	13.21	12.67	+0.54	470*.	1	14.48	13.85	+0.63
427...	1	13.26	11.80	+1.46	471..	4	12.62	12.04	+0.58
428*..	2	14.89	14.30	+0.59	472..	1	12.86	12.34	+0.52
430...	4	13.32	12.88	+0.44	472a.	1	15.60	14.63	+0.97
431...	1	12.55	12.11	+0.44	474*.	1	14.44	14.04	+0.40
431a..	1	16.00	15.28	+0.72	475..	6	14.42	14.40	+0.02
432*..	1	13.64	13.32	+0.32	476..	1	13.16	12.36	+0.80
433...	2	13.79	12.16	+1.63	477*.	2	13.24	12.70	+0.54
436...	4	14.12	13.74	+0.38	480..	5	13.52	13.28	+0.24
437*..	3	11.46	10.69	+0.77	481..	2	13.88	13.78	+0.10
438*..	2	14.84	14.38:	+0.46:	482*.	2	13.49	13.06	+0.43
439...	6	13.64	13.49	+0.15	483..	6	14.32	14.23	+0.09
440*..	5	14.18	14.10	+0.08	484..	4	14.15	13.89	+0.26
442*..	5	14.00	13.82	+0.18	485..	1	14.54	14.10	+0.44
443...	5	13.58	13.32	+0.26	486..	1	12.85	12.45	+0.40
444...	4	14.61	14.25	+0.36	487..	2	12.39	11.59	+0.80
445...	1	12.74	12.42	+0.32	488..	6	14.52	14.51	+0.01
445a..	1	15.24	14.78	+0.46	489..	3	13.40	12.91	+0.49
446...	3	13.58	13.27	+0.31	490*.	3	13.12	12.62	+0.50
447...	2	14.06	13.64	+0.42	490a*	3	14.56	14.08	+0.48
448...	5	14.02	13.94	+0.08	491*.	2	12.33	11.33	+1.00
449...	1	14.30	14.20	+0.10	492..	1	14.10	13.76	+0.34
451...	6	13.09	12.84	+0.25	493..	2	12.55	11.92	+0.63
452...	1	12.65	11.89	+0.76	494..	3	12.32	11.47	+0.85
454...	6	12.66	12.16	+0.50	494a.	3	15.14	14.80	+0.34
455...	6	14.84	14.62	+0.22	495*.	2	12.42	11.59	+0.83

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
496a*	3	14.70	13.87	+0.83	540*	3	12.08	11.02	+1.06
496β*	3	13.14	12.36	+0.78	541	5	14.35	14.16	+0.19
496γ*	3	14.80	13.76	+1.04	542*	2	14.00	13.61	+0.39
497	3	14.82	14.43	+0.39	542a*	2	14.74	14.00	+0.74
498	1	13.21	12.71	+0.50	543	2	15.20	14.86	+0.34
499*	5	13.50	13.22	+0.28	544	3	11.97	11.35	+0.62
501	4	11.96	11.11	+0.85	545*	2	9.3:	8.5	+0.8:
502	2	14.24	14.05	+0.19	546*	4	13.47	12.68	+0.79
503*	2	14.52	14.21	+0.31	548*	4	12.23	11.32	+0.91
504*	3	12.31	11.94	+0.37	551	2	12.62	12.10	+0.52
505*	2	12.80	11.92	+0.88	552	6	14.20	14.28	−0.08
507*	3	13.02	11.81	+1.21	553	2	14.94	14.67	+0.27
508	6	13.23	12.93	+0.30	553a	2	15.77	15.12	+0.65
509*	2	13.28	12.46	+0.82	554	3	13.93	13.84	+0.09
513*	2	14.46	14.22	+0.24	555*	3	14.74	12.91	+1.83
514	2	15.13	14.73	+0.40	556	4	13.66	13.50	+0.16
515	7	14.09	14.08	+0.01	556a	4	16.17	15.08	+1.09
516	3	12.12	11.50	+0.62	557	4	13.56	11.50	+2.06
517	2	13.50	13.07	+0.43	557a	5	15.14	14.85	+0.29
519	4	13.30	12.88	+0.42	557b	5	15.14	14.68	+0.46
519a	4	15.56	15.00	+0.56	558	3	14.42	14.32	+0.10
520	2	15.10	14.43	+0.67	558a	4	16.68	14.68	+2.00
520a	2	15.43	14.86	+0.57	559*	3	15.08	14.38	+0.70
522	2	13.61	13.04	+0.57	560*	3	12.80	11.11	+1.69
523	3	12.25	11.53	+0.72	561*	2	14.78	13.92	+0.86
523a	3	15.39	14.93	+0.46	562	3	14.55	14.28	+0.27
524	2	14.04	13.94	+0.10	563	2	13.12	12.61	+0.51
525*	3	12.08	11.14	+0.94	563a	3	15.48	15.01	+0.47
525a*	3	14.58	14.21	+0.37	564*	3	14.28	13.40:	+0.88:
526	4	13.63	13.06	+0.57	564a*	2	15.36	14.56	+0.80
526a*	4	15.41	14.66	+0.75	564b*	3	15.36	14.52	+0.84
528	7	13.54	13.48	+0.06	565	4	13.66	13.62	+0.04
529	5	13.48	13.36	+0.12	567	3	14.61	14.40	+0.21
532	3	14.10	14.07	+0.03	568*	3	13.31	12.16	+1.15
532a	4	15.47	14.76	+0.71	568a*	3	15.46	14.67	+0.79
533	3	13.44	13.22	+0.22	569	6	13.26	12.96	+0.30
533a	3	15.93	14.92	+1.01	570*	4	13.20	12.87	+0.33
533′	2	14.22	14.12	+0.10	570a	5	15.38	14.69	+0.69
534*	2	12.67	11.27	+1.40	571*	3	13.16	12.16	+1.00
536	5	14.58	14.22	+0.36	572	4	13.33	12.94	+0.39
536a	5	15.37	14.80	+0.57	572a	4	15.54	14.96	+0.58
538	2	12.60	12.05	+0.55	573	3	13.13	12.74	+0.39
538a	2	14.39	14.33	+0.06	573a	3	15.34	14.88	+0.46
539	3	13.29	11.31	+1.98	574	3	12.94	12.41	+0.53
539a*	3	15.50	14.57	+0.93	576*	3	12.39	11.65:	+0.74:

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
576a*.	3	14.30	14.02:	+0.28:	614a*	5	16.12	14.12	+2.00
577...	4	12.22	11.44	+0.78	615..	4	13.92	13.70	+0.22
579a*.	4	14.72	14.81	−0.09	616..	6	13.89	13.66	+0.23
579β*.	4	14.38	13.73	+0.65	617..	8	13.25	12.78	+0.47
579a..	4	15.32	14.90	+0.42	618..	7	14.09	13.84	+0.25
580*..	3	14.34	14.06	+0.28	619..	5	13.86	13.36	+0.50
581...	4	11.85	11.08	+0.77	620..	7	13.33	13.06	+0.27
581a..	4	14.56	14.35	+0.21	621..	7	14.10	13.80	+0.30
582...	4	14.35	14.32	+0.03	622..	4	14.39	13.78	+0.61
583...	6	13.84	13.69	+0.15	622a.	4	15.36	14.78	+0.58
585...	3	14.05	13.76	+0.29	623..	5	13.83	13.62	+0.21
586...	4	13.73	13.60	+0.13	623a.	5	16.00	15.10	+0.90
588...	3	15.14	14.88	+0.26	624..	4	14.04	14.00	+0.04
588a..	4	15.30	15.00	+0.30	630..	5	13.54	13.52	+0.02
589...	4	15.11	14.73	+0.38	631..	7	12.46	11.94	+0.52
590...	3	13.04	11.20	+1.84	633..	8	13.37	13.18	+0.19
591...	7	14.64	14.15	+0.49	634..	6	13.46	13.22	+0.24
592*..	5	14.22	13.88	+0.34	635..	5	13.23	13.01	+0.22
593...	3	13.88	13.58	+0.30	637..	7	15.08	14.69	+0.39
593a..	3	15.47	14.92	+0.55	638..	5	14.88	14.72	+0.16
594...	4	13.28	12.80	+0.48	639..	6	14.64	14.44	+0.20
596...	6	14.43	14.10	+0.33	640*.	6	13.12:	13.04:	+0.08:
597...	7	12.78	11.92	+0.86	643..	6	15.08	14.60	+0.48
598...	4	13.02	11.46	+1.56	644..	5	14.30	13.94	+0.36
598a..	4	15.40	14.73	+0.67	645..	7	15.15	14.67	+0.48
599...	5	12.40	11.80	+0.60	647..	5	12.66	11.98	+0.68
600...	5	14.19	13.97	+0.22	648..	6	13.54	13.16	+0.38
600a..	5	15.96	15.00	+0.96	649*.	5	14.77	14.32	+0.45
601*..	8	11.69	10.68:	+1.01:	650..	6	14.54	14.58	−0.04
602...	4	14.02	13.80	+0.22	652..	8	13.16	12.81	+0.35
603...	7	12.83	12.14	+0.69	653..	6	13.52	13.12	+0.40
604...	4	13.34	12.94	+0.40	654..	6	13.36	11.92	+1.44
604a..	3	15.68	14.90	+0.78	655..	6	14.97	14.62	+0.35
606...	4	13.72	13.31	+0.41	656..	7	14.31	13.67	+0.64
606a..	4	16.38	15.57	+0.81	657..	6	14.49	14.04	+0.45
607...	4	13.48	13.34	+0.14	658..	6	14.28	13.93	+0.35
608...	5	14.45	14.23	+0.22	660..	6	14.39	14.14	+0.25
609...	5	13.21	11.20	+2.01	662..	7	12.47	12.00	+0.47
610...	4	13.48	13.27	+0.21	663..	6	11.90	11.22	+0.68
610a..	4	15.52	14.84	+0.68	664..	7	12.79	12.30	+0.49
610b..	4	15.26	14.54	+0.72	666..	8	13.45	12.01	+1.44
611...	4	13.39	12.95	+0.44	668..	7	14.82	14.64	+0.18
612...	6	14.82	14.58	+0.24	670..	7	13.92	13.60	+0.32
613...	4	14.15	13.99	+0.16	671..	7	13.66	13.40	+0.26
614*..	5	15.48	14.62	+0.86	672..	6	12.50	12.12	+0.38

TABLE IV—*Continued*

No.	Region	Pg. Mag.	Pv. Mag.	Color-Index	No.	Region	Pg. Mag.	Pv. Mag.	Color-Index
674...	7	13.28	11.84	+1.44	680..	7	13.58	12.84	+0.74
675...	7	13.62	13.30	+0.32	681..	6	14.90	14.51	+0.39
677...	8	15.17	14.76	+0.41	682..	9	13.05	12.72	+0.33
679...	6	15.10	14.66	+0.44	685..	7	14.92	14.62	+0.30

NOTES TO THE CATALOGUE

Star

233. Components not separated on isochromatic plates.

248. Components not separated on Seed 27 plates.

421*a*. A close double, components unequal.

461. Chosen as central star by Stratonoff.

462. Position of this star adopted as center of cluster; it is 32″ south of No. 461.

470. Close double, components not separately measured.

545. This bright star, *B.D.*$-6^\circ 4929$, is No. 1 in the catalogues of Lamont and Helmert, and is the star upon which the Mount Wilson plates were centered. It is a minute of arc distant from the center of the cluster, and probably is not physically connected with the group. The star is too bright for accurate measurement on photographs taken with the full aperture of the 60-inch reflector. The visual magnitude according to the *B.D.* catalogue is 8.8, and, according to Lamont and to Helmert, 9.0 and 8.6, respectively. Stratonoff's photographic magnitude, which, however, is based on visual standards, is 8.6. No proper motion has been detected in this or in any of the reference stars (Stratonoff, *op. cit.*, p. 24).

555. May be variable. About half a magnitude fainter July 8, 1916, than June 7, 1915. The value of the color-index given in the table is of the right order.

560. Close double, components not separately measured.

570. Close double, components not separately measured.

640. A close double; components equally bright, both photographically and photo-visually but not separately measured.

The arrangement of the material in Tables VI and VII follows that of the analogous tables in preceding contributions.[1] The 14 stars fainter than photo-visual magnitude 15.0 are excluded from Table VII. In the region covered, which is a square 6′ on a side with star No. 462 as center, all stars brighter than 15.0 are included. The cluster itself is apparently limited to a little more than half of this area, and is also limited chiefly to the brighter magnitudes here considered. The density of the stars fainter than magnitude 14.0 or 14.5 is little, if any, greater in the cluster than outside, and there

[1] Tables XII and XIII of *Mt. Wilson Contr.*, No. 116, and Tables VII and VIII of No. 117.

TABLE V

POSITIONS OF POSTSCRIPT STARS

Star	R.A. 1900	Decl. 1900	Star	R.A. 1900	Decl. 1900
226*a*........	$18^{h}45^{m}31^{s}.8$	−6° 21′ 12″	494*a*.......	$18^{h}45^{m}44^{s}.0$	−6° 24′ 39″
226*b*........	32.8	21 4	496α.......	43.9	21 51
226*c*........	32.3	20 30	496β.......	44.0	21 52
233α........	32.6	23 18	496γ.......	44.1	21 53
233β........	32.6	23 20	519*a*.......	45.3	24 46
248*a*........	33.4	22 55	520*a*.......	44.4	23 28
249*a*........	34.9	21 58	523*a*.......	45.9	22 16
249*b*........	34.7	22 4	525*a*.......	45.2	24 33
251*a*........	34.3	24 28	526*a*.......	45.5	21 38
258*a*........	34.5	24 16	532*a*.......	46.0	21 57
266*a*........	34.7	21 32	533*a*.......	45.1	24 18
266*b*........	34.8	21 37	536*a*.......	46.5	21 29
267*a*........	34.8	23 37	538*a*.......	45.5	23 2
267*b*........	34.2	23 55	539*a*.......	46.0	24 19
273*a*........	35.3	23 10	542*a*.......	45.9	23 25
287*a*........	37.0	24 13	553*a*.......	46.3	22 57
293*a*........	36.2	22 5	556*a*.......	46.6	22 6
296*a*........	37.0	21 39	557*a*.......	45.9	25 6
310*a*........	37.3	23 40	557*b*.......	46.4	25 3
323*a*........	37.6	21 55	558*a*.......	47.5	24 26
324*a*........	38.2	22 6	563*a*.......	46.8	22 57
330*a*........	37.2	24 9	564*a*.......	46.6:	23 16
336*a*........	38.2	22 32	564*b*.......	46.8	23 18
338*a*........	37.7	21 23	568*a*.......	46.8	23 47
345*a*........	38.2	24 54	570*a*.......	48.1	21 41
348*a*........	38.7	21 32	572*a*.......	47.6	21 43
376*a*........	38.9	24 46	573*a*.......	47.3	22 58
381*a*........	39.7	24 41	576*a*.......	47.5	22 34
394*a*........	40.9	22 48	579α.......	47.5	24 47
397*a*........	40.5	21 17	579β.......	47.6	24 45
411*a*........	41.3	22 53	579*a*.......	47.1	24 50
419*a*........	41.3	24 50	581*a*.......	47.5	23 26
420*a*........	41.9	22 11	588*a*.......	48.6	22 42
421*a*........	41.9	24 22	593*a*.......	48.2	23 50
431*a*........	41.2	23 29	598*a*.......	48.6	22 47
445*a*........	41.8	22 54	600*a*.......	48.4	21 24
456*a*........	42.9	22 4	604*a*.......	48.3	23 30
460*a*........	42.6	24 19	606*a*.......	49.5	22 17
461*a*........	42.2	22 31	610*a*.......	50.0	22 46
462*a*........	42.6	23 17	610*b*.......	50.2	22 33
466*a*........	42.9	23 25	614*a*.......	49.4	21 54
468*a*........	43.9	24 57	622*a*.......	50.0	23 36
472*a*........	43.2	23 30	623*a*.......	50.2	24 48
490*a*........	44.1	24 15			

TABLE VI

Frequency of Colors and Magnitudes

Distance from Center	Limits of Pv. Mag.	Color-Class							All Colors
		b	*a0–a4*	*a5–a9*	*f*	*g*	*h*	*m*	
0′.0 to 1′.0	11.0–11.5								
	11.5–12.0				2	1	1	1	5
	12.0–12.5			1	7	1	1	1	11
	12.5–13.0			1	4				5
	13.0–13.5		1	1	3				5
	13.5–14.0		2	7	1				10
	14.0–14.5		4	2	2				8
	14.5–15.0		1*	3	4	1			9
	>15.0			2	2				4
1.0 to 2.0	11.0–11.5				5	5	3	3	16
	11.5–12.0				6		1	1*	8
	12.0–12.5			1	5				6
	12.5–13.0			2	14				16
	13.0–13.5		1	7	2				10
	13.5–14.0		9	11	4				24
	14.0–14.5		6	12	3	1	1		23
	14.5–15.0		1	11	23	2		1	38
	>15.0			1	2	2			5
2.0 to 3.0	11.0–11.5				2			1†	3
	11.5–12.0			2	3		2		7
	12.0–12.5			1	3				4
	12.5–13.0			4	4				8
	13.0–13.5		4	8	4		1	1	18
	13.5–14.0	1	5	7					13
	14.0–14.5	2	7	9	1				19
	14.5–15.0	1	9	11	10				31
	>15.0				2	3			5
>3.0	11.0–11.5				1			2‡	3
	11.5–12.0				2	2	1		5
	12.0–12.5				4		1		5
	12.5–13.0			3	2				5
	13.0–13.5		2	6			1		9
	13.5–14.0		3	3	2			1	9
	14.0–14.5		3	4	1				8
	14.5–15.0	1	4	2	2				9
	>15.0								0
All of cluster	11.0–11.5				8	5	3	6	22
	11.5–12.0			2	13	3	5	2	25
	12.0–12.5			3	19	1	2	1	26
	12.5–13.0			10	24				34
	13.0–13.5		8	22	9		2	1	42
	13.5–14.0	1	19	28	7			1	56
	14.0–14.5	2	20	27	7	1	1		58
	14.5–15.0	2	15	27	39	3		1	87
	>15.0			3	6	5			14
	All magnitudes....	5	62	122	132	18	13	12	364

* Color-Index = +2.06. † Color-Index = +2.01. ‡ Contains one star with color-index of +2.05.

TABLE VII

DISTANCE AND COLOR CLASS

Distance from Center	Quantity	Color-Class						All Colors
		*a*0–*a*4	*a*5–*a*9	*f*	*g*	*k*	*m*	
	No. stars...	1	5	8 (1)	2	1	1	18
0′.0 to 0′.5	Av. Pv. mag.	14.20	13.68	12.83	13.50	11.80	11.83	13.10
	Av. C. I....	+0.10	+0.30	+0.51	+0.88	+1.46	+1.63	+0.59
	No. stars...	7	10	15 (4)	1	1	1	35
0.5 to 1.0	Av. Pv. mag.	14.03	13.92	13.21	11.59	12.18	12.16	13.47
	Av. C. I....	+0.12	+0.31	+0.53	+0.80	+1.36	+1.63	+0.45
	No. stars...	6	23	24 (8)	4	2	3	62
1.0 to 1.5	Av. Pv. mag.	14.00	13.91	13.29	12.21	12.82	11.33	13.41
	Av. C. I....	+0.10	+0.30	+0.56	+0.91	+1.40	+1.86	+0.53
	No. stars...	11	20	38 (12)	5	3	2*	79
1.5 to 2.0	Av. Pv. mag.	13.87	14.02	13.46	13.26	11.55	13.09	13.56
	Av. C. I....	+0.11	+0.30	+0.55	+1.04	+1.48	+2.03	+0.53
	No. stars...	13†	19	14 (5)	0	2	1	49
2.0 to 2.5	Av. Pv. mag.	14.12	13.85	13.39	....	12.56	11.20	13.68
	Av. C. I....	+0.11	+0.27	+0.55	....	+1.43	+2.01	+0.39
	No. stars...	16‡	23	13 (2)	0	1	1	54
2.5 to 3.0	Av. Pv. mag.	14.15	13.72	13.30	....	11.92	13.44	13.71
	Av. C. I....	+0.06	+0.28	+0.49	....	+1.44	+1.73	+0.32
	No. stars...	7§	13	11 (5)	2	2	2‖	37
3.0 to 3.5	Av. Pv. mag.	14.29	13.61	12.76	11.92	12.65	11.10	13.21
	Av. C. I....	+0.09	+0.29	+0.57	+0.98	+1.44	+2.00	+0.53
	No. stars...	6	5	3 (0)	0	1	1	16
>3.5	Av. Pv. mag.	13.93	13.67	13.26	....	12.01	13.50	13.58
	Av. C. I....	+0.14	+0.28	+0.46	....	+1.44	+1.88	+0.44
	No. stars...	67	118	126 (38)	14	13	12	350
All of cluster	Av. Pv. mag.	14.07	13.83	13.27	12.68	12.20	12.04	13.50
	Av. C. I....	+0.10	+0.29	+0.54	+0.96	+1.44	+1.88	+0.467

* Color-indices are +2.00 and +2.06.
† Contains one star with color-index of −0.16.
‡ Contains three stars with color-indices of −0.08, −0.04, and −0.02.
§ Contains one star with color-index of −0.10.
‖ Contains one star with color-index of +2.05.

is no evidence on any of the photographs of the existence of the highly concentrated background of faint stars that is typical of truly globular clusters. In fact, the cluster is apparently a condensation, on a rich background, of 150 or 200 stars whose magnitudes are between 11.0 and 14.5. The distribution (number of stars per square minute for successive regions) is illustrated in Fig. 1.

No conspicuous difference in average color or average magnitude is evident between the center of the cluster and the outermost region. In this similarity of cluster stars to the non-cluster stars

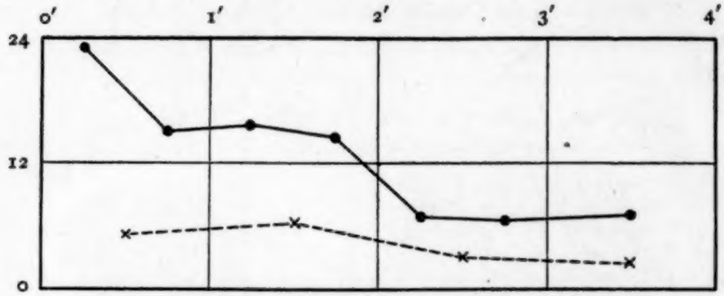


FIG. 1.—Distribution of stars in Messier 11. Full line, all stars brighter than pv. mag. 15.0; broken line, stars between pv. mag. 14.0 and 15.0.

Ordinates: Number of stars per square minute
Abscissae: Distance from center of the cluster

in the immediate neighborhood, the conditions here are like those in Messier 67, the open cluster discussed in the foregoing paper. The result supports the hypothesis that open clusters are an integral part of the galactic system and in most characteristics differ little from their environment.

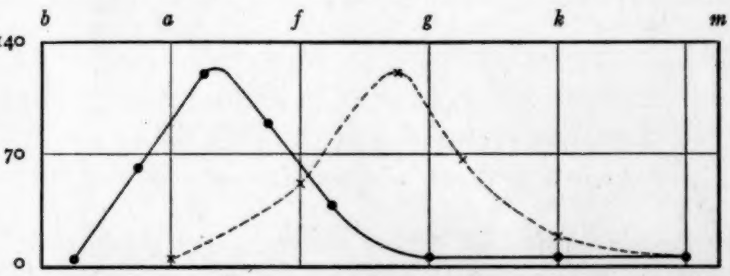


FIG. 2.—Relative frequency of colors. Full line, Messier 11; broken line, Messier 67. Ordinates: Numbers of stars. Abscissae: Color classes.

Curves of relative frequency of colors are shown in Fig. 2. The data for Messier 67 are taken from Table VII of *Contribution* No. 117. The distribution of color-classes is seen to be totally different in the two open groups. The color-index of maximum frequency is about $0^{M}.6$ bluer for the cluster in the star clouds than

for the one of higher galactic latitude. Moreover, the small color-indices in Messier 11 are not confined to the cluster itself and its immediate surroundings, but occur throughout that region of the sky. The point will be treated further in the following paper of this series, which deals with the colors in several parts of the surrounding galactic clouds.

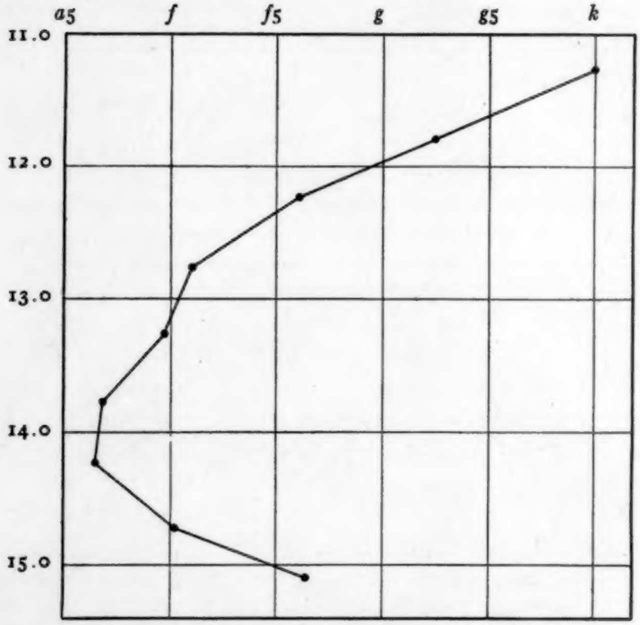


FIG. 3.—Relation between color and luminosity in Messier 11

Ordinates: Color classes. Abscissae: Apparent magnitudes

In the same paper will appear a discussion of the decrease of brightness with decreasing color-index—a point wherein Messier 11 bears a resemblance to the Hercules cluster. For the present the average color-index for successive intervals of magnitude is shown in Table VIII, and a plot of the results appears in Fig. 3. To supplement the lower part of the curve, provisional color-indices of a number of faint stars have been determined from Plates 3235 and 3236. For 34 objects the average photo-visual magnitude is 15.34, and the average color-index +1.03; but because of a

selection, uncertain in amount, of some stars peculiarly red and others peculiarly blue, the result for this group of fainter stars is not of high weight.

TABLE VIII

MAGNITUDE AND AVERAGE COLOR

Interval of Pv. Mag.	Number of Stars	Mean Pv. Mag.	Mean C.I.
11.0 to 11.5..	22	11.26	+1.20
11.5 to 12.0..	25	11.80	+0.90
12.0 to 12.5..	26	12.22	+0.64
12.5 to 13.0..	34	12.75	+0.44
13.0 to 13.5..	42	13.25	+0.39
13.5 to 14.0..	56	13.75	+0.27
14.0 to 14.5..	58	14.23	+0.26
14.5 to 15.0..	87	14.71	+0.41
>15.0..	14	15.09	+0.66

SUMMARY

1. A catalogue has been made of the photographic and photovisual magnitudes of 458 stars within four minutes of arc of the center of the open galactic cluster Messier 11, which is situated in one of the densest star clouds in the sky. Although considerable difficulty was experienced in obtaining satisfactory series of plates, the average probable error of a resulting color-index does not exceed a tenth of a magnitude.

2. It seems probable that the cluster proper is composed of not more than 200 stars, nearly all of which are brighter than magnitude 14.5. The diameter of the group appears to be less than five minutes of arc.

3. In absolute values and frequency of color-indices, the stars in the cluster are not unlike the stars in its neighborhood; but they average half a magnitude bluer than those of the same magnitude in and near the open cluster Messier 67. The galactic latitude of the latter is $+34^\circ$, of the former -3°.

4. The presence of small color-indices among the faint stars in this low galactic region must have a significant bearing on the extent of the Milky Way.

MOUNT WILSON SOLAR OBSERVATORY
November 1916

THE COLOR-CURVE OF XZ CYGNI[1]

By MARTHA BETZ SHAPLEY

The progressive change of spectral type throughout the period of light-variation, recently found for Cepheid variables, denotes a synchronous and continuous change of color-index. As detailed studies of spectral variation will be possible for only the brighter stars, the color-curves afford the best means of inferring the changes in spectral class of the fainter Cepheids, particularly of those with half-day periods. As yet only one or two variables of this type have definitive color-curves. A further contribution to the subject is made in the present paper, which deals with the photographic and photo-visual light-curves of XZ Cygni,[2] a cluster-type variable with unusually rapid change of light.

Ninety-five multiple-exposure photographs, of which 50 are on Seed "27" and 45 on Cramer "Instantaneous Iso" plates, were obtained by Mr. Seares and Mr. Shapley, and kindly placed at my disposal. The plates are described in Tables I and II. An asterisk after the number in the first column indicates that in addition to the exposures on the field of the variable, the plate was also exposed on the field of the North Polar Standards to determine the magnitudes of the comparison stars. For such plates the number after the comma in the sixth column refers to the exposures on the Pole. The magnitude in the fifth column is the mean deduced from all the exposures on one plate. The light-elements used in computing phases are those of Enebo as corrected by Martin and Plummer:[3]

$$\text{Maximum} = \text{J. D. } 2417201.2542 + 0^{d}.466586 \cdot E$$

which represent the Mount Wilson observations with sufficient accuracy.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 128.

[2] XZ Cygni = B.D. + 56°2257 = A.Oe. 19376 = A.G. Helsingfors 10573. The position for 1900.0 from *Hamburger Astronomische Abhandlungen*, 1, No. 3, p. 95, 1909, is: R.A. = $19^{h}\ 30^{m}\ 24^{s}.92$, Dec. = 56°10′22″.8.

[3] *Monthly Notices*, 74, 226, 1914.

Because of the great difference in the brightness of the available comparison stars, diaphragms, as indicated in the seventh column, were employed to make the images comparable with those of the variable. The duration of exposure was one minute for the Seed "27" and three minutes for the Isochromatic plates.

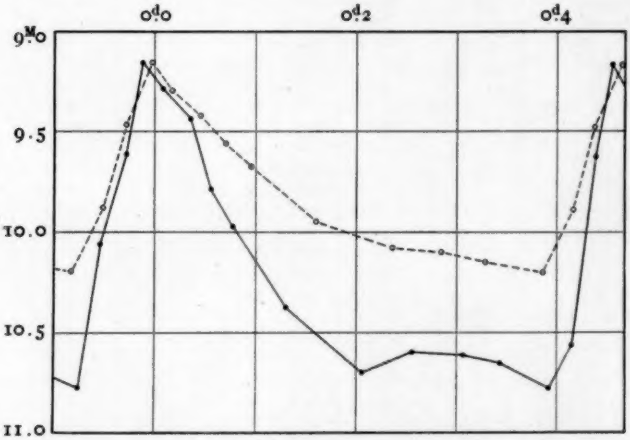


FIG. 1.—Mean photo-visual (broken line) and photographic light-curves of XZ Cygni

The methods of measurement and reduction were those generally employed in photometry with the 60-inch reflector.[1] The North Polar plates yield photographic and photo-visual magnitudes for the comparison stars as shown in Table III. For these determinations, Plates 1904, 1926, 1939, 2860, and 1911, 1927, 1954, 2861 were not used because of the distorted images of polar stars.

The observations in Tables I and II were combined into normals in order of phase, generally four in a group, giving the data of Table IV. Plotting the mean magnitudes against phase, we have the light-curves represented in Fig. 1. Whatever irregularities exist in the shape of the maximum at different epochs and in the color-change are necessarily obscured in a mean curve; and the period of XZ Cygni is too long to permit a determination of the complete color-variation at a single epoch.

[1] Seares, *Mt. Wilson Contr.*, No. 80; *Astrophysical Journal*, **39**, 307, 1914.

TABLE I

Photographic Observations of XZ Cygni

Plate Number	Date	Julian Day and Gr. H.M.T.	Phase	Pv. Mag.	No. Exposures	Diaphragms	Normal Number
1715....	1914 July 19	2420333.901	0ᵈ.455	9.22	5	14, 9, 6	13
1717....		.915	0.002	9.45	5	32, 14, 9, 6	1
1719....		.932	0.019	9.45	5	14, 9, 6	1
1721....		.949	0.036	9.82	5	32, 14, 9	2
1723....		.964	0.051	10.04	5	32, 14, 9	3
1725....		.979	0.066	9.90	5	32, 14, 9	4
1727....		.993	0.080	9.93	5	14	4
1851....	Aug. 26	2420371.656	0.417	10.23	8	14, 9	11
1853....		.674	0.435	9.49	8	14, 9	12
1855....		.693	0.454	9.14	8	14, 9	13
1857....		.712	0.006	9.13	8	14, 9	1
1859....		.732	0.026	9.30	8	14, 9	2
1861....		.751	0.045	9.42	8	14, 9	3
1863....		.769	0.063	9.97	8	14, 9	3
1865....		.787	0.081	10.12	8	14, 9	4
1867....		.805	0.099	10.29	8	14, 9	5
1869....		.825	0.119	10.25	8	14, 9	5
1871....		.849	0.143	10.36	8	14, 9	5
1873....		.867	0.161	10.64	8	14, 9	5
1875....		.885	0.179	10.83	8	14, 9	6
1877....		.906	0.200	10.78	8	14, 9	6
1879....		.924	0.218	10.78	8	14, 9	6
1904*...	Sept. 18	2420394.799	0.230	10.42	8, 2	14, 9	6
1906....		.817	0.248	10.68	8	14, 9	7
1908....		.837	0.268	10.62	8	14, 9	7
1910....		.855	0.286	10.62	8	14, 9	8
1926*....	Sept. 19	2420395.844	0.342	10.58	2, 2	14, 9	9
1939*....	Sept. 20	2420396.737	0.302	10.57	8, 2	14, 9	8
1941....		.758	0.323	10.52	8	14, 9	8
1943....		.775	0.340	10.63	8	14, 9	9
1945....		.793	0.358	10.66	8	14, 9	9
1947....		.813	0.378	10.69	8	14, 9	10
1949....		.836	0.401	10.69	8	14, 9	10
1951....		.853	0.418	10.32	8	14, 9	11
1953....		.871	0.436	9.47	8	14, 9	12
2027....	Sept. 22	2420398.758	0.457	9.12	8	14, 9	13
2029....		.775	0.007	9.12	8	14, 9	1
2031....		.795	0.027	8.95	8	14, 9	2
2033....		.813	0.045	9.70	8	14, 9	2
2035....		.832	0.064	9.72	8	14, 9	3
2037....		.854	0.086	9.93	8	14, 9	4
2043....	Sept. 23	2420399.623	0.388	11.07	4	32, 14	10
2045....		.638	0.403	10.84	4	32, 9	11
2047....		.652	0.417	10.85	4	32, 9	11
2051....		.682	0.447	9.89	4	32, 6	12
2860*....	1915 Nov. 30	2420832.624	0.398	10.68	2, 2	14	10
3214*....	1916 July 7	2421052.703	0.248	10.52	3, 2	14, 9	7
3215*....		.708	0.253	10.60	3, 2	14, 9	7
3232*....	July 8	2421053.704	0.316	10.72	3, 2	14	8
3234*....		.714	0.326	10.72	2, 2	14	9

TABLE II

PHOTO-VISUAL OBSERVATIONS OF XZ CYGNI

Plate Number	Date		Julian Day and Gr. H.M.T.	Phase	Pg. Mag.	No. Exposures	Diaphragms	Normal Number
1716....	1914	July 19	2420333.909	0^d463	9.28	3	32, 14, 9	12
1718....			.924	0.011	9.30	3	32, 14, 9	1
1720....			.941	0.028	9.32	4	32, 14, 9, 6	1
1724....			.972	0.059	9.52	3	32, 14	3
1726....			.987	0.074	9.62	3	40, 32, 14	3
1728....			2420334.000	0.087	9.73	3	32	4
1852....		Aug. 26	2420371.665	0.426	9.62	8	32, 14	10
1854....			.682	0.443	9.32	8	32, 14	11
1856....			.703	0.464	9.16	8	32, 14	12
1858....			.721	0.016	9.34	8	32, 14	1
1860....			.742	0.036	9.11	8	32, 14	2
1862....			.760	0.054	9.61	8	32, 14	2
1864....			.778	0.072	9.50	8	32, 14	3
1866....			.796	0.090	9.62	8	32, 14	4
1868....			.814	0.108	9.68	8	32, 14	4
1870....			.834	0.128	9.84	8	32, 14	5
1872....			.858	0.152	10.04	8	32, 14	5
1874....			.876	0.170	9.95	8	32, 14	5
1876....			.894	0.188	9.96	8	32, 14	5
1878....			.915	0.209	9.99	8	32, 14	6
1905....		Sept. 18	2420394.808	0.239	10.05	4	32, 14	6
1907....			.826	0.257	10.08	4	32, 14	6
1909....			.846	0.277	10.05	4	32, 14	7
1911*....			.864	0.295	10.06	4, 2	32, 14	7
1927*....		Sept. 19	2420395.857	0.355	10.16	2, 2	32, 14	9
1940....		Sept. 20	2420396.745	0.310	10.03	4	32, 14	7
1942....			.767	0.332	10.07	4	32, 14	8
1944....			.783	0.348	10.08	4	32, 14	8
1948....			.826	0.391	10.29	4	32, 14	9
1950....			.844	0.409	10.00	4	32, 14	10
1952....			.862	0.427	9.78	4	32, 14	11
1954*....			.879	0.444	9.32	4, 2	32, 14	11
2028....		Sept. 22	2420398.767	0.466	9.04	4	32, 14	12
2030....			.786	0.018	9.22	4	32, 14	1
2032....			.804	0.036	9.43	4	32, 14	2
2034....			.823	0.055	9.54	4	32, 14	2
2036....			.844	0.076	9.59	4	32, 14	3
2038....			.864	0.096	9.67	4	32, 14	4
2044....		Sept. 23	2420399.631	0.396	10.20	4	32, 14	9
2046....			.647	0.412	10.06	4	32, 14	10
2861*....	1915	Nov. 30	2420832.628	0.402	10.13	2, 1	14	9
3213*....	1916	July 7	2421052.699	0.244	10.19	2, 2	14	6
3216*....			.712	0.257	10.26	2, 2	14	7
3231*....		July 8	2421053.701	0.313	10.28	3, 4	60, 14	8
3233*....			.711	0.323	10.17	1, 2	14	8

From Fig. 1 it is apparent at once that the photographic range is much in excess of the photo-visual. At maximum the star is of about the same brightness in blue and in yellow light, indicating a spectrum of perhaps Ao. At minimum the difference in brightness

reaches nearly 0.6 mag., and at the corresponding point the spectral class is probably F4 or F5. Three spectrograms made by Mr. Adams show an actual range of A0 to A8; but the length of the exposures on his plates (two to two and one-half hours) tends to obscure and lessen the range, though not affecting the type at maximum.

TABLE III

MAGNITUDES OF COMPARISON STARS

Star	B.D. Number	Photographic Magnitude	Photo-visual Magnitude
1*	....	11.43±0.03	10.96±0.02
2	+56°2259	11.88±0.03	10.09±0.07
3	+56°2261	7.79±0.03	6.98±0.07
4†	....	12.16±0.01	10.64±0.03

* Position, 1855: R.A. = $19^h\ 29^m\ 46^s$, Dec. = +56° 4′.5.
† Position, 1855: R.A. = $19^h\ 28^m\ 55^s$, Dec. = +56° 5′.4.

TABLE IV

MEAN LIGHT-CURVES

Photographic			Photo-visual		
No.	Mean Phase	Mean Mag.	No.	Mean Phase	Mean Mag.
1	0ᵈ008	9.29	1	0ᵈ018	9.30
2	.034	9.44	2	.045	9.42
3	.056	9.79	3	.070	9.56
4	.078	9.97	4	.095	9.68
5	.130	10.38	5	.160	9.95
6	.206	10.70	6	.237	10.08
7	.254	10.60	7	.285	10.10
8	.307	10.61	8	.329	10.15
9	.342	10.65	9	.386	10.20
10	.391	10.78	10	.416	9.89
11	.414	10.56	11	.438	9.47
12	.439	9.62	12	0.464	9.16
13	0.455	9.16			

The photometric data concerning maximum and minimum light and color of XZ Cygni are as follows:

	Pg. Mag.	Pv. Mag.	C.I.	Color-Class
Maximum	9.16	9.16	0.00	*a0*
Minimum	10.78	10.20	0.58	*f4*
Range	1.62	1.04	0.58	....

The range and shape of the photographic curve are in fair agreement with the results of Martin and Plummer.[1]

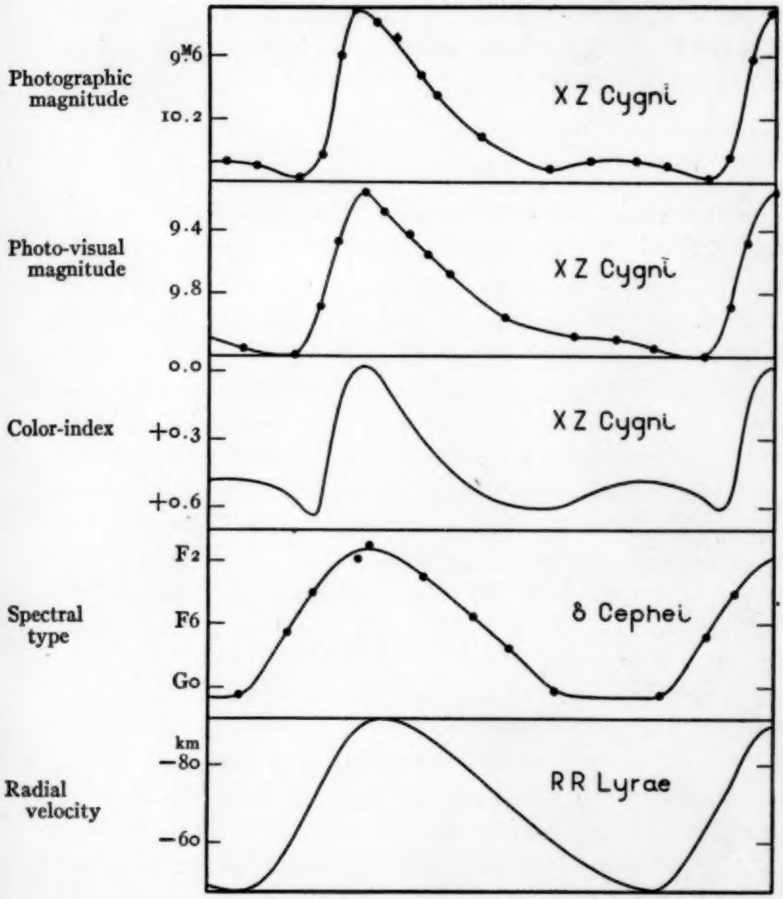


FIG. 2.—Five aspects of Cepheid variation

In conclusion, Fig. 2 is presented as an interesting summary of the various periodic changes which Cepheid variables undergo from one epoch of maximum light to another: (1) variation in photographic light, (2) variation in photo-visual light, (3) variation

[1] *Monthly Notices*, **74**, 230, 1914.

in color, (4) variation in spectrum,[1] (5) variation in radial velocity.[2] Since data relating to (4) and (5) are not available for XZ Cygni, the curves from other objects of the Cepheid class have been inserted for comparison. There is a striking similarity in the shape and fluctuations of all the curves. It is impossible not to conclude that all these phenomena are due to one underlying cause.

Mount Wilson Solar Observatory
January 1917

[1] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916.

[2] *Lick Observatory Bulletins*, **7**, 143, 1913.

THE APPLICATION OF THE PLANE GRATING TO THE DETERMINATION OF THE INDEX OF REFRACTION OF A GAS, WITH VALUES FOR AIR FROM λ 2500 TO λ 6500[1]

By ROBERT WILLIAM DICKEY

INTRODUCTION

There are many methods applicable to the determination of the index of refraction of a gas for wave-lengths in the visible spectrum and the near ultra-violet spectrum. Any method, to be of universal use, must give results in the extreme ultra-violet which are accurate to the same degree as those in the visible region. It was the purpose of the present investigation to study the use of the plane grating in extending as far as possible into the ultra-violet region the dispersion curve of a gas. The test of any method is the value of the results derived by it, consequently the index of refraction of air has been obtained for different wave-lengths between 2493 and 6412 A. A careful study of the possibilities and difficulties of the method has been made.

HISTORICAL

As the problem is an old one, it has been undertaken by a great many investigators. A few of the most important methods will be reviewed along with the results found.

Ketteler,[2] using a Jamin interferometer, observed the fringe-shift for definite change in pressure of the gas. His difficulty was that of obtaining efficient sources of monochromatic light, although the method gave absolute values. He obtained the index of refraction for yellow sodium light for air, carbon dioxide, hydrogen, sulphur dioxide, and cyanogen.

L. Lorenz,[3] working with a similar apparatus, determined the refractive indices of air freed of moisture and carbon dioxide, and

[1] Dissertation, Johns Hopkins University.

[2] *Poggendorf's Annalen*, **124**, 390–406, 1865.

[3] *Annalen der Physik und Chemie*, N.F. **11**, 70, 1880.

of oxygen and nitrogen for sodium and lithium light. Mascart,[1] making use of Talbot bands, measured the absolute value of the index of refraction of air and chlorine for sodium light and the relative values for the four cadmium lines. Perreau[2] used the actual tubes of Mascart, but discarded the method involving Talbot bands and obtained a banded spectrum by analyzing white light which had passed through a Jamin refractometer. Kayser and Runge[3] placed a hollow prism between a photographic plate and a large concave grating and measured the displacement of various spectral lines when the pressure of the air in the prism was changed. They worked between $\lambda = 5630$ and $\lambda = 2360$ with a pressure-change of 10 atmospheres.

Rentschler[4] used a Fabry and Perot interferometer and a concave grating and determined the indices of refraction of air, nitrogen, oxygen, carbon dioxide, and carbon monoxide from $\lambda = 5769$ to $\lambda = 3341$. The interferometer plates were silvered, and owing to the low reflecting power of silver between $\lambda = 2280$ and $\lambda = 3341$ Rentschler's results were not carried below $\lambda = 3341$. Among the recent publications of work in the visible spectrum is that of C. and M. Cuthbertson, who used a Jamin refractometer and determined the indices of refraction of air, oxygen, nitrogen, hydrogen,[5] neon,[6] krypton, xenon, helium, and argon[7] between $\lambda = 6563$ and $\lambda = 4861$. Quite recently an investigation of this problem has been carried out by Miss Howell,[8] using a Fabry and Perot interferometer (with plates nickeled by cathode discharge), in connection with a quartz spectrograph. She has extended the dispersion-curve of air to $\lambda = 2652$ and those of hydrogen, oxygen, and carbon dioxide to $\lambda = 2753$. The advantage of this method is that it is not necessary to count the number of interference bands going by for a change of pressure, thus making the method applicable to the ultra-violet region.

[1] *Comptes rendus*, **86**, 321, 1878.

[2] *Annales de Chimie et de Physique*, **7**, 289, 1896.

[3] *Annalen der Physik*, **50**, 293, 1893.

[4] *Astrophysical Journal*, **28**, 345, 1908.

[5] *Proc. Roy. Soc.*, **83**, 151, 1909.

[6] *Ibid.*, **83**, 149, 1909.

[7] *Ibid.*, **81**, 440, 1908.

[8] *Physical Review*, **6**, 81, 1915.

The method here described consists in determining the index of refraction of a gas by finding the ratio of the wave-lengths in vacuum and in the gas by means of a Rowland plane grating, using a Littrow type of mounting. The general idea involved is not new, for Osborne and Lester[1] used this method for the determination of the index of refraction of a liquid. Its application to a gas is new and there are many details in which this method differs from the one mentioned above.

THEORETICAL CONSIDERATIONS

If v_0 is the velocity of light-waves in the pure ether, v, the velocity in a definite gas, then the index of refraction of the gas with reference to the ether is defined as the ratio of v_0 to v. Calling the index of refraction n, we have

$$n=\frac{v_0}{v}=\frac{\nu\lambda_0}{\nu\lambda}$$

where ν is the frequency corresponding to the wave-length λ.

$$\therefore n=\frac{\lambda_0}{\lambda} \text{ and } n-1=\frac{\lambda_0-\lambda}{\lambda}=\frac{\Delta\lambda}{\lambda}\,. \tag{1}$$

Since λ is known, it is necessary to measure $\Delta\lambda$ for a change in pressure of the gas. To accomplish this, the fundamental principle of the grating is made use of. The condition under which we obtain a bright line of the mth order, when white light is allowed to fall upon the grating, is that the path difference between rays reflected from adjacent grooves shall be $m\lambda$, where λ is the wave-length of the line in question. The equation for the Littrow mounting is

$$m\lambda=a(\sin\theta+\sin i) \tag{2}$$

where θ is the angle of incidence of the light, and this is kept constant, and i is the angle of the diffracted beam. i differs very slightly from θ. If the grating is surrounded by a gas, let the path difference for some particular line in the spectrum be $m\lambda_1$. Then when the same grating is placed in vacuum, the path difference between adjacent grooves for the same line will be $m\lambda_2$, according to equation (1), where λ_2 is greater than λ_1. Since θ is constant in

[1] *Ibid.*, **35**, 210, 1912.

equation (2), the change in λ is accompanied by a change in i and a shift of the line is observed. If a photographic plate is placed in the focal plane of the observing telescope, this shift can be recorded by making two exposures, one with the grating in vacuum and the second with the grating in the gas, keeping the grating fixed during the two exposures.

APPARATUS AND METHOD

The Littrow form of spectrometer (Fig. 1) was used. To avoid instrumental shifts due to the possible relative motion of the parts of the apparatus, it was mounted as rigidly as possible. Two 3-inch iron I-beams, 300 cm in length, were fastened together and parallel to each other so that the flanges gave a firm horizontal surface in which to mount the various parts. At one end a cast-iron base-plate 22 cm in diameter and 3.2 cm thick was mounted and insulated from the I-beams by hard rubber supports so as to prevent as much as possible the transference of heat to or from the plate. At the center of this plate the grating-holder was attached with the proper adjustments. Just behind the grating-holder the connection to the pump and outside air was made as shown in Fig. 1. At the other end of the beams an iron plate, carrying the slit, reflecting prism, and photographic plate-holder, was fastened. The collimator lens was mounted on a slide fastened between the I-beams. In focusing for any definite region of the spectrum the collimator lens alone was moved. A heavy brass bell-jar, 12.5 cm inside diameter, 14 cm deep, with walls 1 cm thick and a heavy flange at the bottom, covered the grating. The front was arranged so that a quartz window could be inserted and made air-tight. The opening for the window was 2.5×4 cm. Through a hole in the top of the bell-jar a thermometer reading to tenths of a degree was inserted.

A Rowland 2.5-inch grating, ruled with 15,000 lines to the inch, was used. A grating with bright third and fourth orders was selected. On account of the high coefficient of expansion of speculum metal, every precaution was taken to keep the temperature of the grating constant during a series of exposures. A heating coil wound on the inside walls of a large wooden box (40×55×75 cm),

surrounding the bell-jar, helped to keep the temperature constant. Then the massiveness of the bell-jar tended to oppose any rapid changes in the temperature of the inside. This problem of temperature has proved a very serious trouble, and it will be discussed more fully in a later paragraph.

The entire optical system, consisting of a small condensing lens *C*, a totally reflecting prism *P*, the collimator lens *L*, and the plane parallel plate *W*, was made of quartz. As a source of light the

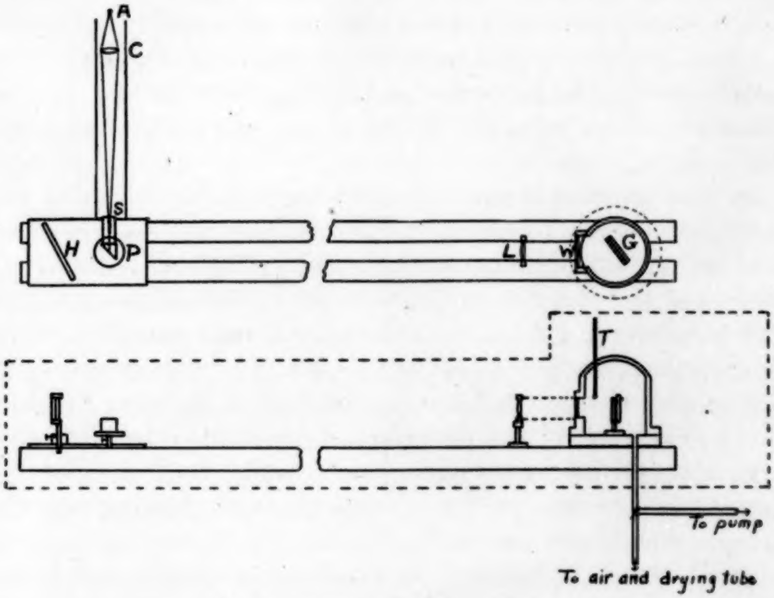


Fig. 1.—Arrangement of apparatus

Pfund iron arc, operated on 110 volts with 5 or 6 amperes, was used. An image of the arc was thrown on the slit *S* by means of the condensing lens *C*, and a larger image was projected on the wall of the room at a distance of 5 meters so as to make certain that the same part of the arc was being used in a series of exposures. The observations were recorded on photographic plates.

The general procedure was to pump out the bell-jar until the pressure was a small fraction of a millimeter of mercury, take an exposure on the photographic plate, then let in the air through

drying solutions and make a second exposure on the same plate, being very careful not to disturb it between exposures. The greatest difficulty to be overcome was the change in temperature of the grating between exposures, due to a thermodynamic increase or decrease in the temperature of the surrounding air when the pressure was increased or decreased. The resultant change in the temperature of the air itself is negligible so far as the density of the air is affected, but the high coefficient of expansion of the metal of the grating causes change in the grating space. This in turn causes a considerable shift of the spectrum lines for even a small fraction of a degree change in temperature. Upon the reduction of the pressure in the bell-jar by about one atmosphere, a sudden drop of about 1° C. was indicated by the thermometer whose bulb was inside the inclosure.

If one could tell when the grating reached a steady state, the difficulty would be overcome, but it is evident that there is a considerable lag of temperature in the grating with reference to the thermometer. A thermo-couple with one set of junctions insulated and imbedded in a brass block attached to the grating was used in an attempt to determine the steady state, but this had to be abandoned owing to the heat conducted to or away from the junction by the leads. An attempt was then made to avoid the difficulty by lowering the temperature of the air to that of liquid air before letting it into the bell-jar, but this method failed and was discarded.

Although this error due to temperature changes could not apparently be done away with, it was decreased to a certain extent by the filling of all the unnecessary space in the bell-jar with pieces of metal of high specific heats and large absorbing surfaces.

Finally, the scheme of lowering and raising the pressure of the air at intervals of twenty minutes and taking all exposures three minutes before and three minutes after letting the air into the bell-jar was adopted. In this way the error due to temperature-changes will be the same for all observations. The foregoing cycle of operations was repeated several times before any observations were made. As the total change in the shift due to thermodynamic changes was only a few thousandths of a millimeter, it was impossible to determine the actual value to a reasonable degree of accu-

racy by taking exposures at different times before and after letting the air into the bell-jar. The only course left open was to take the mean value of the index of refraction of air for some line in the visible spectrum and refer all observations to this one value. The exposures for different regions of the spectrum varied from 15 seconds to 5 minutes, so it was necessary to adopt the three-minute intervals mentioned above.

The spectrum of the third order was used between $\lambda=6400$ and $\lambda=4300$, of the fourth order from $\lambda=4100$ to $\lambda=2700$, and of the third order for the extreme ultra-violet observations. Even though the overlapping spectra were not in focus, much trouble was experienced by the fog produced on the plates. By the proper selection of absorbing solutions and screens this was avoided to a certain extent.

In the design of the apparatus any mechanism for setting the grating without removing the bell-jar was purposely avoided, for it was thought that such an arrangement might cause an instrumental shift in the grating when the pressure in the inclosure was changed. The grating was set at the desired angle by means of a telescope and scale, the grating being used as a mirror, after the sides of the constant temperature box and the bell-jar had been removed. When the bell-jar was replaced over the grating it was necessary to make the plane-parallel quartz window perpendicular to the incident light from the lens. This was done by adjusting the bell-jar until the image of the part of the incident light reflected from the front surface of the quartz plate fell in the center of the photographic plate and just below the spectrum. The location of this image plays an important part in one of the corrections which will be discussed later.

The air was freed from moisture by passing it through sulphuric acid and over phosphorous pentoxide. The pressure in the bell-jar was measured by a mercury manometer, the readings being reduced to mercury at $0°$ C. Readings of temperature were made to $0.^\circ05$ C. and of pressure to one-tenth of a millimeter.

RESULTS

Observations were made at intervals of about 200 A, three or four plates being made for each region. This gave a means of

checking the measurements taken from a single plate. In every case tried the values from two different plates were found to agree.

The position of the lines on the plates and their corresponding shifts were measured by a traveling microscope, the screw of which was made in the same way as the screws for the Rowland ruling machines. Measurements were made to 1/1000 mm. In cases where the lines on any one plate were numerous, only those best adapted to measurement were selected. Each line and its corresponding shift were measured on an average of six or seven times and the mean of the results taken.

Position correction.—In the preliminary experiments it was noticed that the shift was not only horizontal, but vertical to a small degree. In looking for an explanation of this, it was found that a correction, depending upon the distance of the line from the

FIG. 2

center of the plate, had to be made. This correction can be deduced with the aid of Fig. 2. The only essential parts to be considered are the quartz plate W, which serves as a boundary between the outside air and the inclosure, and the photographic plate. If, when the pressure in space 2 is the same as that in space 1, the path of the ray from the grating G to the photographic plate is ABC, its path, when the pressure in 2 is less than that in 1, will be ABD owing to the deviation of the ray as it passes from a rarer to a denser medium. At the center of the optical system this will not occur, for the ray is normal to the quartz plate W.

Let n' = the index of refraction of space 1 with reference to space 2 at the observed temperature and pressure. This value of n' is derived from measurements at the center of the photographic plate where no correction is necessary.

Let l = distance ED, or distance of line from center of plate

Let θ = tilt of photographic plate

Then $CD=\left(\frac{n'l\cos\theta}{EB}-\frac{l\cos\theta}{EB}\right)EB$, approximately, $=(n'-1)\,l\cos\theta$

On account of this error the measured shifts taken from the half of the plate toward the red end of the spectrum are too small, and those of the other half too large. In referring the shifts to the center of the plate, the foregoing correction had to be added to or subtracted from the measured values of the "red" end or "blue" end respectively of the plate. Obviously it was very necessary to determine the center of the photographic plate, and this was done by means of the image of the slit reflected from the front surface of the quartz window. This has been discussed in the section on the apparatus and method.

The values of the shifts, after the foregoing correction had been made, were reduced to standard conditions. As a result of many experiments the following relation between the index of refraction of a substance and its density,

$$\frac{n-1}{\rho}=\text{constant},$$

has been well established by many investigators. Further, the shift for any definite wave-length is such that

$$\frac{n-1}{\Delta\lambda}=\text{constant}.$$

Hence, putting

$$\Delta\lambda=s,$$

it follows that

$$\frac{s}{\rho}=\text{constant}.$$

Or, expressing ρ in terms of pressure and absolute temperature,

$$\frac{sT}{p}=\text{constant}.$$

Hence, if s_1 is observed at temperature T_1 and pressure p_1 (expressed in millimeters of mercury), the value of s, which would have been observed if the temperature had been 0° C. and the pressure 760 mm, is given by the formula

$$\frac{s273}{760}=\frac{s_1(273+t_1)}{p_1}$$

or

$$s=s_1\frac{760}{p_1}\,\frac{273+t_1}{273}.$$

As the shifts reduced to standard conditions were given in millimeters, it was then necessary to reduce them to angstrom units. To do so it was necessary to compute the dispersion for each line of the plate. The positions of three lines, one at the center of the plate, x_0, and one near each end of the plate, x_1 and x_2 respectively, were carefully measured. It was assumed that any two wave-lengths with their corresponding positions obeyed the following relation,

$$\lambda_0-\lambda=A(x_0-x)+B(x_0-x)^2,$$

in which A and B are constants, x_0 the position of λ_0 on the plate, and x the position of any other line λ. λ_0 is taken as near as possible to the center of the plate. To obtain the values of A and B the three lines mentioned above were used. Differentiating the expression, we have

$$\frac{d\lambda}{dx}=A+2B(x_0-x).$$

Then if x_0 is selected, the dispersion at any other point on the plate indicated by x is known from the above relations. All the shifts were then multiplied by the value of $\frac{d\lambda}{dx}$ for the corresponding value of x. The resultant values are the respective changes in the wave-lengths of the different lines of the spectrum as the pressure is changed by the amount observed. According to equation (1), $n-1=\frac{\Delta\lambda}{\lambda}$; hence the values of $n-1$ could be calculated. The wave-lengths of the lines were taken from Kayser's *Handbuch.*

A specimen of the results is given in Table I. This is one of the seventeen tables obtained. In this table column I gives the relative positions of the lines on the photographic plate; column II, the corresponding wave-lengths as taken from Kayser's *Handbuch;* column III, the mean values of the shifts measured; column IV, the shifts corrected for the distance of the line from the center of

TABLE I

I x(mm)	II λ(A)	III S_2(mm)	IV S_1(mm)	V S(mm)	VI $\frac{d\lambda}{dx}\frac{(A)}{mm}$	VII $\Delta\lambda$(A)	VIII $n-1$	IX
$\lambda_1$256.188	4199.25	0.5501	0.5338	0.5943	2.1285	1.2650	0.0003012	
258.483	4204.15	.5506	.5349	.5956	2.1276	1.2671	.0003014	
259.964	4206.87	.5509	.5355	.5962	2.1256	1.2673	.0003012	For
263.039	4213.82	.5505	.5360	.5968	2.1232	1.2671	.0003007	λ=4219.52
264.243	4216.33	.5508	.5366	.5984	2.1223	1.2699	.0003012	$n-1$=0.0003011
265.740	4219.52	.5514	.5376	.5986	2.1211	1.2696	.0003009	
277.985	4245.43	.5540	.5433	.6049	2.1115	1.2772	.0003009	
288.707	4268.93	.5565	.5486	.6108	2.1032	1.2847	.0003009	
290.574	4271.95	.5562	.5488	.6111	2.1017	1.2843	.0003006	
295.643	4282.59	.5575	.5514	.6139	2.0978	1.2879	.0003007	
297.103	4285.60	.5567	.5510	.6135	2.0966	1.2863	.0003001	
301.239	4294.32	.5587	.5541	.6170	2.0934	1.2916	.0003008	
306.647	4305.63	.5590	.5558	.6188	2.0892	1.2928	.0003003	For
307.818	4308.09	.5598	.5569	.6201	2.0883	1.2950	.0003006	λ=4315.29
$\lambda_3$311.262	4315.29	.5605	.5585	.6219	2.0856	1.2970	.0003006	$n-1$=0.0003005
319.000	Center of plate							
321.803	4337.24	.5624	.5631	.6270	2.0774	1.3025	.0003003	
329.352	4352.90	.5637	.5664	.6306	2.0715	1.3063	.0003001	
337.578	4369.95	.5661	.5709	.6357	2.0651	1.3128	.0003004	
340.558	4376.11	.5662	.5718	.6367	2.0628	1.3134	.0003001	
344.242	4383.71	.5668	.5734	.6384	2.0600	1.3151	.0003000	
354.535	4404.95	.5700	.5792	.6449	2.0519	1.3232	.0003004	
356.323	4408.58	.5697	.5794	.6451	2.0505	1.3238	.0003001	For
363.212	4422.74	.5713	.5828	.6489	2.0451	1.3271	.0003001	λ=4408.58
365.530	4427.50	.5726	.5847	.6510	2.0433	1.3302	.0003004	$n-1$=0.0003001
367.145	4430.79	.5713	.5838	.6500	2.0420	1.3273	.0002996	
$\lambda_2$372.878	4442.52	0.5736	0.5876	0.6543	2.0376	1.3332	0.0003001	

t_1=24°.6 C.; p_1=744.1 mm.; angle of tilt of camera =12°; third order of grating.

the plate; column V, the shifts reduced to standard conditions; column VI, the dispersion in angstrom units per millimeter as measured on the photographic plate; column VII, the shifts in angstrom units, or rather the change in wave-lengths due to the change in pressure; and column VIII the values of $n-1$ as calculated by equation (1). The calculation of the index of refraction from one line alone is not very reliable, for the physical characteristics of the line may be such as to influence the results. For this reason the practice of determining the final values from a number of lines has been adhered to. This is shown in column IX. The values of $n-1$ were reduced to one particular line by taking as nearly as possible the same number of evenly spaced lines on each side of a central line, and taking the mean of the values. This practice is permissible in the region of the spectrum where the dispersion is comparatively small, but in the ultra-violet the separate values of $n-1$ were reduced to one particular line by the use of Kayser and Runge's dispersion formula, and the mean of these reduced values was taken as a final value. Table I represents one of the better sets of observations. In some cases values of $n-1$ had to be determined from a less number of lines.

Since the temperature correction for the shifts was not determined, it was necessary to select some iron line whose index of refraction for air has been well determined by other observers, and refer all values to this line. For this purpose the line $\lambda=4315.29$ was selected. The index of refraction was calculated from the dispersion formulae of Kayser and Runge,[1] Rentschler,[2] and Miss Howell,[3] the values obtained for $n-1$ being 0.0002961, 0.0002959, and 0.0002958. The mean of these is 0.0002959. The value obtained in the present investigation was 0.0003005, thus giving a difference of 0.0000046, which will be called the temperature correction. From the following discussion it will be seen that this correction is constant for all regions of the spectrum; and the corrected values of the index of refraction for air for all wave-lengths is found by subtracting this correction from the computed values.

[1] *Annalen der Physik*, **50**, 293, 1893.

[2] *Astrophysical Journal*, **28**, 345, 1908.

[3] *Physical Review*, **6**, 81, 1915.

The formula for the Littrow type of mounting of the plane grating is

$$m\lambda = a(\sin\theta + \sin i)$$

where m is the order of the spectrum, λ the wave-length of the spectrum line, a the grating space, θ the angle the incident light makes with the normal to the grating, and i the angle of the refracted beam. For the present discussion θ and i can be considered as remaining constant. Suppose a temperature-change in the grating occurs, then the expansion produces a change, δa, in the grating space. The corresponding change in the wave-length, $\delta\lambda$, is given by the relation

$$m\delta\lambda = \delta a(\sin\theta + \sin i).$$

Dividing this expression by the one above, we have

$$\frac{\delta\lambda}{\lambda} = \frac{\delta a}{a} = \text{constant},$$

if the change δa is a constant in every case. The exposures were carried out under exactly the same conditions so that δa would be a constant. Therefore the correction to $n-1$ for temperature changes is a constant for all values of λ.

The indices of refraction of air for the different wave-lengths obtained in this manner are given in Table II. The first column contains the values of the wave-lengths, the second, the observed values of $n-1$, and the third the corrected values of $n-1$. In the fourth column are recorded the number of lines observed in determining $n-1$ for the specified wave-length, λ.

The dispersion-curve is shown in Fig. 3. Curve I indicates the values obtained from the observed values of the shifts of the lines on the photographic plate, while curve II represents the values of $n-1$ referred to the known value for the line $\lambda = 4315.29$.

It is customary to express dispersion-curves in terms of the Cauchy formula,

$$n-1 = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4},$$

although this is of no theoretical importance. It is given here so that the results can be compared with those of other observers. According to Cuthbertson (*loc. cit.*)

$$n-1=10^{-7}\left(2885.4+\frac{13.38}{\lambda^2}+\frac{.50}{\lambda^4}\right);$$

Kayser and Runge (*loc. cit.*) find

$$n-1=10^{-7}\left(2878.7+\frac{13.16}{\lambda^2}+\frac{.316}{\lambda^4}\right);$$

Rentschler (*loc. cit.*) finds

$$n-1=10^{-7}\left(2903+\frac{3.80}{\lambda^2}+\frac{1.23}{\lambda^4}\right);$$

Miss Howell (*loc. cit.*) finds

$$n-1=10^{-7}\left(2881.7+\frac{11.83}{\lambda^2}+\frac{.46}{\lambda^4}\right)$$

where λ is expressed in thousandths of a millimeter. In the present investigation the values of the constants a, b, and c in the Cauchy formula were obtained by the method of least mean squares, and the following formula was found:

$$n-1=10^{-7}\left(2878.12+\frac{11.59}{\lambda^2}+\frac{0.551}{\lambda^4}\right).$$

This is in good agreement with the formula of Kayser and Runge and that of Miss Howell.

DISCUSSION OF RESULTS

A remarkably smooth dispersion-curve was obtained, only one point failing to touch the curve. This was at $\lambda=4879.39$, and the reason for this can be found in the physical characteristics of the lines in this region. As they were broad and not well defined, accurate settings on them were almost impossible. For the converse reason the values below $\lambda=4500$ are more accurate.

The magnitude of the shift is also a determining factor in the accuracy of the results as well as the sharpness of the lines. To increase the magnitude of the shift, higher orders of the grating were used as far as possible, in keeping with the intensity of the

TABLE II

λ (A)	$n-1$ (Observed)	$n-1$ (Corrected)	No. Lines Used in Determining $n-1$
6411.90......	0.0002956	0.0002910	3
6230.93......	.0002960	.0002914	4
5615.89......	.0002965	.0002919	8
5406.02......	.0002968	.0002922	8
5328.21......	.0002971	.0002925	4
5269.70......	.0002971	.0002925	3
5202.49......	.0002972	.0002926	9
5139.64......	.0002975	.0002929	1
4879.39......	.0002986	.0002940	7
4710.46......	.0002988	.0002942	2
4618.95......	.0002991	.0002945	7
4517.70......	.0002997	.0002951	9
4408.58......	.0003001	.0002955	7
4315.29......	.0003005	.0002959	12
4219.52......	.0003011	.0002965	7
4118.70......	.0003015	.0002969	18
3941.03......	.0003023	.0002977	12
3867.35......	.0003027	.0002981	10
3651.61......	.0003043	.0002997	12
3575.60......	.0003049	.0003003	3
3450.46......	.0003058	.0003012	12
3292.73......	.0003076	.0003030	11
3037.51......	.0003107	.0003061	9
2727.64......	.0003165	.0003119	14
2664.77......	.0003184	.0003138	4
2613.91......	.0003205	.0003159	11
2567.01......	.0003228	.0003182	8
2493.34......	0.0003268	0.0003222	4

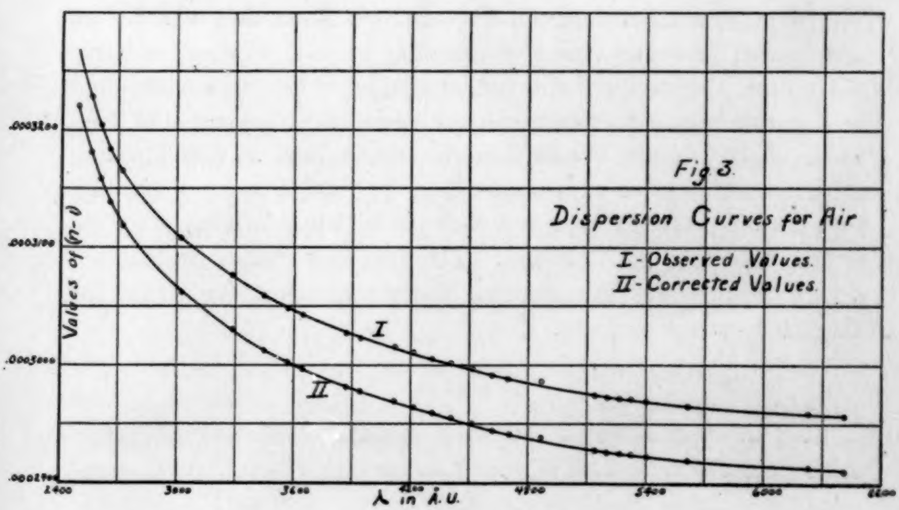


Fig. 3

lines. The large angle of tilt of the photographic plate in the extreme ultra-violet made it possible to use the third order in this region and obtain the same magnitude of shift. The shifts varied from 0.45 mm to 0.92 mm, the particular value depending upon the order used and the region of the spectrum.

The dispersion-curve was not extended below $\lambda=2493$, owing to the arrangement of the apparatus. The opening in the bell-jar was such as greatly to reduce the amount of the grating space used and consequently to cut down the intensity of the spectral lines. From a consideration of the number of separate measurements which are associated with a single value of $n-1$ and the good agreement of these measurements, it is safe to say that the fourth significant figure is determined to a fair degree of accuracy. It can be relied upon to be correct to within several units.

As far as the method is concerned, there seems to be only one serious difficulty—the effect of temperature-changes as the pressure of the gas is changed. It is believed that, with several improvements in the present apparatus, the value of this temperature correction can be determined and consequently the absolute value of $n-1$. The author hopes to continue this work in the near future, using other gases. In the present form this method is applicable to gases which do not injure the grating and for which a single value of the index of refraction is known.

Among the advantages of this method the following may be mentioned: first, any source of light may be used which gives fairly sharp lines, the number being immaterial provided there are enough in any one region to determine the dispersion; second, the final value of $n-1$ is not obtained from observations on a single line; third, values of $n-1$ can be obtained for wave-lengths of all lines that can be photographed in a reasonable length of time using an ordinary grating spectrometer. This last fact makes the method a very desirable one in extending into the extreme ultra-violet the dispersion-curves of gases.

SUMMARY

1. The application of the plane grating to the determination of the index of refraction of gases has been thoroughly studied and

the only serious difficulty is found to be one due to changes of temperature.

2. The dispersion-curve of air has been obtained from

$$\lambda=6412 \text{ to } \lambda=2493.$$

3. The accuracy of the corrected values of the index of refraction of air agrees with those obtained by other methods.

In conclusion I wish to thank Dr. Anderson, who suggested the present problem and whose interest, help, and constant encouragement have been invaluable.

I am indebted to Professor Ames for his interest and advice throughout the course of the work. I wish also to thank Dr. Pfund for his advice and helpful suggestions.

Washington and Lee University
Lexington, Va.
March 1917

A STATISTICAL STUDY OF CERTAIN SOLAR PHENOMENA

By OLIVER J. LEE

The frequent disappointment in following a group of spots or flocculi to the limb of the sun, and in finding either no eruption at all or else merely a few series of small jets where prominences of considerable size and interest had been expected, has led the writer to examine the H calcium plates taken with the Rumford spectroheliograph in the last thirteen years to ascertain whether or not there is any basis for such expectancy. In the first examination of 3200 solar negatives of more recent dates the writer was ably assisted by Mr. Max H. Petersen, of the University of Wisconsin, who was a research scholar at this observatory in the summer of 1916.

The object has been to determine the total number of prominences and the number of prominences probably connected with spots, flocculi, and filaments.

To aid in tracing a point on the disk to its corresponding position on the east or west limb, or vice versa, a chart was made, on the basis of Fox's velocities and gradient of polar retardation as derived from calcium flocculi (see *Report of the Seventeenth Annual Meeting of the American Astronomical Society, 1914*), which gave at a glance the distance of a point from the limbs of the sun in terms of days, for all except very high heliographic latitudes. This chart, with suitable lines for adjusting the inclination of the solar axis, was transferred photographically to a plate, which was mounted on a working desk of illuminated ground glass and was used for features on the solar disk not contiguous to the limbs.

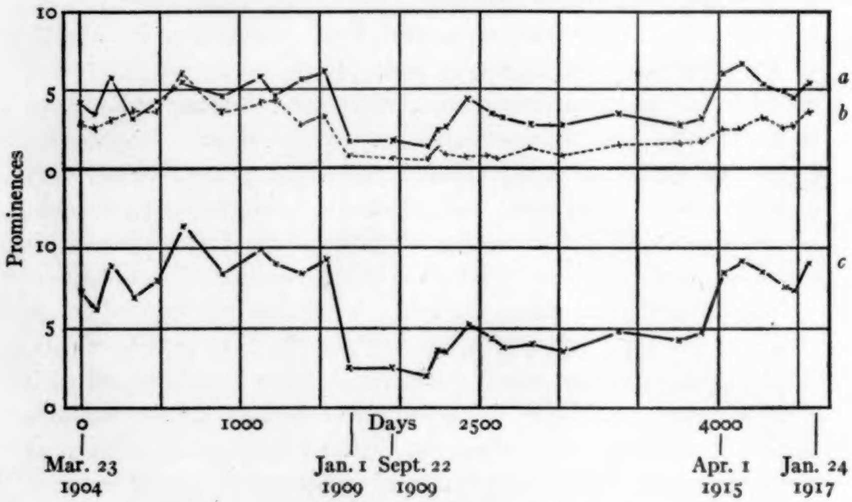
Prominences occurring between $\pm 45°$ of solar latitude and the poles have been counted separately from those lying within the usual range of spots and major flocculi. A center or base of eruption has been made the unit in obtaining numbers. Thus a group of connected prominences has a weight equal to the number of bases

apparently contributing to it, and a fan-shaped bundle of jets emanating from one point has been counted as one prominence. Care has been taken not to count the same prominence on successive days, although a patently new eruption taking the place of one spent has been uniformly counted. The relatively small number of distinct typical forms of prominences enables the observer, in a surprisingly large number of cases, to tell from one plate whether a given prominence is actually on the limb. If photographs of a given prominence taken on successive days be available, it is easy to see whether it has passed the limb or is approaching it.

In tabulating the cases of heliographic connection between prominences and features on the disk the following rules have been observed: (*a*) A prominence anywhere within or contiguous to the field of flocculi surrounding a spot or group of spots has been counted separately as connected with both the spot and the flocculi. Hence, in Table I, if the number of connections of spot and prominence be subtracted from the corresponding number of connections of flocculus and prominence, the result is the number of prominences found to be related directly to flocculi in which no spot is visible. (*b*) A few strong filaments or dark flocculi have been credited with relation to a prominence, if such a one appeared at the position-angle of the former, even when the filament was visible as much as three to four days from the limb and could not be traced up to it because of cloudy weather or because the plates were of lower level, which of course do not show any except the strongest filaments. Such a filament has been counted as occurring on both disk and limb.

The present examination covers the plates taken between March 23, 1904, and January 24, 1917, a period of 4568 days. It covers almost completely the time of the last maximum of solar activity and runs well into the present one. The admirable records of frequencies and profile areas of prominences of the Kodaikanal Observatory since September 1, 1903, as well as the records maintained at various European observatories, give ample data for determining the maxima and minima of prominence activity. It was thought desirable, however, to derive the frequency-curves from the present material for the sake of uniformity of treatment. Probably no two observers would obtain quite the same counts

without conference. The diagrams below give the mean frequencies by groups of 30 to 40 observing days for prominences occurring (*a*) between the $\pm 45°$ parallels of heliographic latitude; (*b*) between these circles and the poles, called for convenience medial and polar prominences, respectively; (*c*) all prominences. With curve (*c*) in hand it is easy to divide the total time interval into two nearly equal parts so that the relations desired may be derived separately for times of smaller and greater prominence activity. The interval



MEAN DAILY FREQUENCY OF PROMINENCES, 1904–1917
a medial prominences, *b* polar prominences, *c* all prominences

between January 1, 1909, and April 1, 1915, has been taken as comprising the former period; the time up to January 1, 1909, and that after April 1, 1915, taken together, as the latter.

In Tables I and II are exhibited the principal results obtained in this investigation. With their headings and subheadings they are self-explanatory. In both Tables I and II the percentages given for prominences connected with filaments are referred to the total number of prominences, both medial and polar, that have been observed, inasmuch as filaments seem to occur indifferently on any part of the solar disk. The percentages given for spots and flocculi are each referred to the total number of medial prominences only, for the obvious reason mentioned above.

The dark absorption markings discovered by Hale and Ellerman[1] on their hydrogen and calcium plates and called by them simply

TABLE I

For the Period of Maximum Frequency

Total number of medial prominences	2867
Total number of polar prominences	1904
Filaments on solar limb or traceable to it	53
Filaments on solar disk	105

Prominences	Connected with					
	Spots		Flocculi		Filaments	
	No.	Percentage	No.	Percentage	No.	Percentage
Very small	20	0.7	76	2.6	10	0.2
Small and medium	103	3.6	243	8.5	18	0.4
Large	43	1.5	108	3.8	19	0.4
Total	166	5.8	427	14.9	47	1.0

TABLE II

For the Period of Minimum Frequency

Total number of medial prominences	1201
Total number of polar prominences	437
Filaments on solar limb or traceable to it	25
Filaments on solar disk	20

Prominences	Connected with					
	Spots		Flocculi		Filaments	
	No.	Percentage	No.	Percentage	No.	Percentage
Very small	21	1.7	45	3.7	6	0.4
Small and medium	37	3.1	67	5.6	7	0.4
Large	12	1.0	21	1.8	3	0.2
Total	70	5.8	133	11.1	16	1.0

dark flocculi[2] are visible only when the light from the central portions of the particular line employed is allowed to enter the second

[1] *Publications of the Yerkes Observatory*, **3**, Part I, 1903, p. 19.

[2] These absorption markings occur outside of, as well as within, the regions where the ordinary bright flocculi are seen. M. Deslandres calls them filaments and alignments, and distinguishes the two as follows: the former are sharp and well defined and on calcium plates appear chiefly when K_3 or H_3 is completely isolated from its

slit of the spectroheliograph. It is a matter of surprise that the relatively small dispersion ordinarily used with the Rumford should reveal them as often as it does. It has been evident from a number of disk plates of the present series that steady seeing is of secondary importance. One instance will suffice for illustration. On November 7, 1907, a strong filament appeared on the east limb of the sun in latitude $+28°$ and extending for $15°$ or more in a southwesterly direction toward the equator. It remained visible on six observing days for a total interval of nine days. All the plates show that the second slit was set quite accurately on the center of the H line. Meanwhile the "seeing" ranged from 2 to 4 on the scale of 5 for perfect steadiness.

So far as the results for the filaments are concerned, it is significant to take the ratio of the number of filaments which are connected with prominences to the total number of filaments on the solar limb or traceable to it. The percentages taken in this sense are 89 and 64, respectively, for the two periods. On a great many high-level plates of the solar disk there are vast areas lying within and beyond (that is, poleward from) the regular belts in which spots and flocculi occur, which have a very rough appearance, although no groups of flocculi are visible in them. The calcium structure within such areas is very complex. Possibly we have here the first stages of flocculation combined with a veritable network of the Deslandres alignments of various forms and sizes. Large eruptive prominences have in many cases been observed to rise from such rough regions.

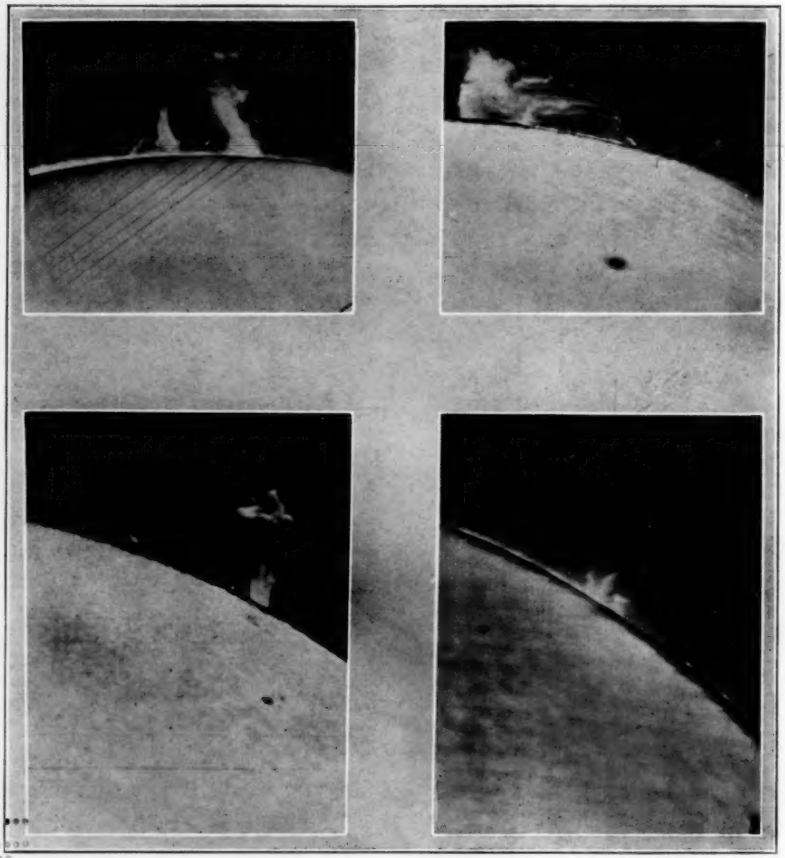
Intensely brilliant spots in areas of flocculi, when traced to the limb, usually show as jets and rarely as prominences of any size.

broader bases K_2 or H_2; the latter are broader and much more diffuse and on plates of high level may be seen as a dusky border along the length of a filament and sometimes as a prolongation of it. Alignments are visible in large numbers on plates taken with smaller dispersion, that is, when light from K_2 and H_2 as well as K_3 or H_3 is certainly entering the second slit, and according to Deslandres they form at times a complete network over large areas on the solar surface.

In this paper I have for convenience used the term filaments to designate both of these phenomena, although it is probable that a majority of the markings I have tabulated would be classed by Deslandres as alignments. Physically the two names perhaps designate the appearances of the same thing photographed at the highest and at an intermediate level, respectively.

PLATE III

a b



c d

PHOTOGRAPHS OF PROMINENCES COMBINED WITH THOSE OF DISK

a. No connection.
Prominence, 1916, September 9, $15^{h}\ 37^{m}$ G.M.T.
Disk, 1916, September 8, $14^{h}\ 25^{m}$.

b. Atmospheric connection.
Prominence, 1916, June 28, $16^{h}\ 9^{m}$ G.M.T.
Disk, 1916, June 27, $15^{h}\ 0^{m}$.

c. Direct connection.
Prominence, 1916, June 2, $15^{h}\ 50^{m}$.
Disk, 1916, June 1, $15^{h}\ 18^{m}$.

d. No connection.
Prominence, 1905, November 14, $16^{h}\ 58^{m}$.
Disk, 1905, November 13, $17^{h}\ 18^{m}$.

At the fifth conference of the International Union for Co-operation in Solar Research, held at Bonn in 1913, it was decided "that the limiting height for statistics of frequency be 30 seconds of arc." On a Rumford plate with the 40-inch telescope this corresponds to 2.8 mm, and since many intensely bright prominences show which are far below this limit, I early decided to include them in this count. In the tables under "very small" are included many such prominences whose height will vary from 30 to 12 seconds of arc. Mere deformations of the chromosphere, such as are frequently observed about spots seen on the limb, have not been counted.

It does not seem necessary to publish the rather voluminous references and notes made in this examination of 5519 plates. They contain the serial number of each prominence plate, the total number of new medial and polar prominences visible on it, the serial numbers of disk plates which show connections (for prominences on the east limb, these are in general plates taken on the same and following days; for prominences on the west limb, the same and preceding days), and finally full notes on the connection which is established.

Figs. *a*, *b*, *c*, and *d* of Plate III are introduced to illustrate the arbitrariness with which even the eruptive prominences do and do not occur where tradition has led us to expect them. From the data shown in the tables it is evident that cases of connection or non-connection between prominences and pronounced features on the solar disk, similar to those shown in the figures, can be multiplied in large numbers. All the prominences exhibited are of the eruptive type, although they differ in degree of activity. All four happened to occur on the west limb of the sun.

Each cut is made from two negatives, one showing the prominence, and a plate of the disk taken on the preceding day. This will enable the reader to see at a glance what is the appearance of the solar surface from which any one of the four groups of prominences rises. The large spot shown in the lower part of Fig. *b* is somewhat over two days from the limb. The large prominence in Fig. *a* appeared at solar latitude $+63°$, the smaller one at $+57°$. Several distinct eruptions took place between these two points on the limb within 2 to 3 days. The simultaneous pair shown in the

figure changed markedly in form within two hours of observation. The larger one is $2'$ in height. There was no spot or floccular area within 40° of this seat of eruptions.

The base of the large eruptions shown in Fig. *b* lies at latitude -30°. The center of the accompanying group of jets is at -20°, and probably coincides with the spot which at the time was less than 1 to $\frac{1}{2}$ days or about $0'.7$ from the limb. The main prominence, $1'.4$ in height, rises out of a clear field on the sun. Because of the apparent interaction between its streamers and the spot shown in the lower part of the figure it was counted as having connection, although the initial eruption probably was independent of the spot. The two prominences shown in Figs. *c* and *d* are of the same eruptive type. The former, a violent eruption, $2'$ in height, appeared in solar latitude $+20^\circ$ and directly over a small spot just visible almost at the base of the prominence, where an hour earlier only a system of faint arching jets about $1'$ high was visible. The latter occurred at -38° and had no connection with any object on the disk.

The results of this investigation may be summarized as follows:

1. Only 236 out of 4068 prominences of all sizes which have been observed between $+45^\circ$ and -45° of solar latitude, or 5.8 per cent, occur in the immediate vicinity of sun-spots.

2. In the same region only 324, or 8.0 per cent, of the prominences occur with flocculi in which no spot was visible during the days of observation.

3. This *infrequency* of connection between prominences and spots and flocculi exists practically to the same degree for times of greater and of less solar activity.

4. Of the 78 filaments which have been observed near the solar limb, 63, or 81 per cent, showed connection with prominences.

5. A considerable number of the large eruptive prominences occur either in unmarked regions of the solar surface or where the surface seems roughened.

There seems to be no reason for preserving the inherited idea that the prominences inevitably or even usually occur in close connection with sun-spots and flocculi.

Yerkes Observatory
March 14, 1917